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Structures for Semantics, assignment 1,
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Exercise 1

(1) Well-formed. $\lambda x \lambda R((M(R(x)) = Q)$
(2) Well-formed. $M(Q) = M(Q)$
(3) Well-formed. $M(Q) = Q$
(4) Well-formed. $\neg(\exists y(P(y) \wedge z \neq y))$
(5) Not well-formed, because z with type e is applied when type $< e, t >$ was expected.
(6) Well-formed. $\exists y1(R(y)(y1))$
(7) Well-formed. $P(y) \wedge \forall y1(Q(y1) \rightarrow R(y)(y1))$
(8) Well-formed. $\exists y(P(j) \rightarrow \exists w(R(j)(w) \wedge B(y) \wedge Q(y)))$

Exercise 2

(a) The functions that can be defined in $D_{\langle e,t \rangle}$ are the members of the set defined as:

$$\prod_{x \in D} \{x\} \times \{0, 1\}$$

To wit: {
$\{\langle d_1, 0\rangle, \langle d_2, 0\rangle, \langle d_3, 0\rangle\}$,
$\{\langle d_1, 0\rangle, \langle d_2, 0\rangle, \langle d_3, 1\rangle\}$,
$\{\langle d_1, 0\rangle, \langle d_2, 1\rangle, \langle d_3, 0\rangle\}$,
$\{\langle d_1, 0\rangle, \langle d_2, 1\rangle, \langle d_3, 1\rangle\}$,
$\{\langle d_1, 1\rangle, \langle d_2, 0\rangle, \langle d_3, 0\rangle\}$,
$\{\langle d_1, 1\rangle, \langle d_2, 0\rangle, \langle d_3, 1\rangle\}$,
$\{\langle d_1, 1\rangle, \langle d_2, 1\rangle, \langle d_3, 0\rangle\}$,
$\{\langle d_1, 1\rangle, \langle d_2, 1\rangle, \langle d_3, 1\rangle\}$
}

(b) With only variables one can only specify the empty and universal functions:

- $\{\langle d_1, 1\rangle, \langle d_2, 1\rangle, \langle d_3, 1\rangle\} =_{def} \lambda x(x = x)$
- $\{\langle d_1, 0\rangle, \langle d_2, 0\rangle, \langle d_3, 0\rangle\} =_{def} \lambda x(x \neq x)$

(c) With the constant for d_3 we can define functions singling out this individual, viz. the d_3 identity function and its negation:

- $\{\langle d_1, 0\rangle, \langle d_2, 0\rangle, \langle d_3, 1\rangle\} =_{def} \lambda x(x = a)$
- $\{\langle d_1, 1\rangle, \langle d_2, 1\rangle, \langle d_3, 0\rangle\} =_{def} \lambda x(x \neq a)$

(d) Using the constant $P = \{\langle d_1, 1\rangle, \langle d_2, 0\rangle, \langle d_3, 1\rangle\}$, we can trivially specify the function described by P, as well as its negation:

- $\{\langle d_1, 1\rangle, \langle d_2, 0\rangle, \langle d_3, 1\rangle\} =_{def} \lambda x P(x)$
- $\{\langle d_1, 0\rangle, \langle d_2, 1\rangle, \langle d_3, 0\rangle\} =_{def} \lambda x \neg P(x)$

Less trivially, we can specify the remaining functions:

- $\{\langle d_1, 0\rangle, \langle d_2, 1\rangle, \langle d_3, 1\rangle\} =_{def} \lambda x(x = a \vee \neg P(x))$
- $\{\langle d_1, 1\rangle, \langle d_2, 0\rangle, \langle d_3, 0\rangle\} =_{def} \lambda x(x \neq a \wedge P(x))$

Exercise 3

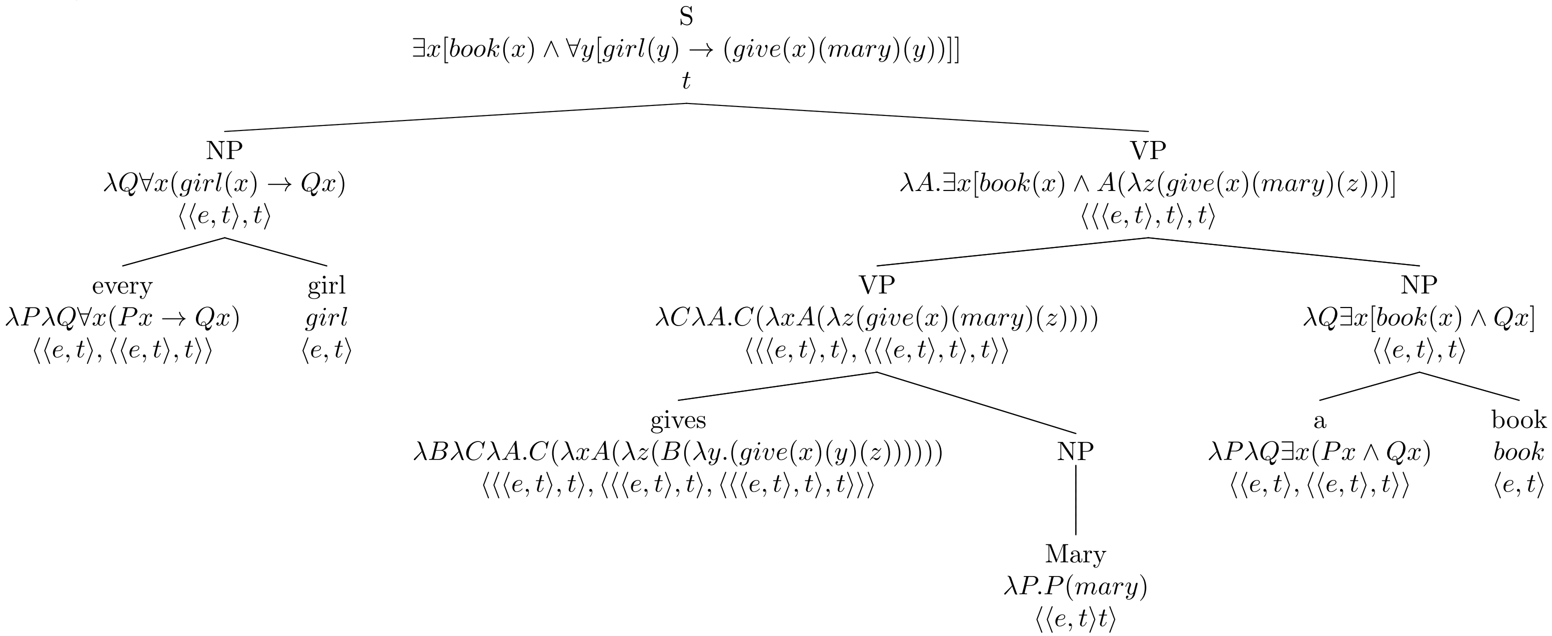
- $\forall x_e \varphi_t =_{def} \lambda\varphi(\lambda x\varphi = \lambda y\top)$
- $\exists x_e \varphi_t =_{def} \neg\forall x \neg\varphi$

Exercise 4

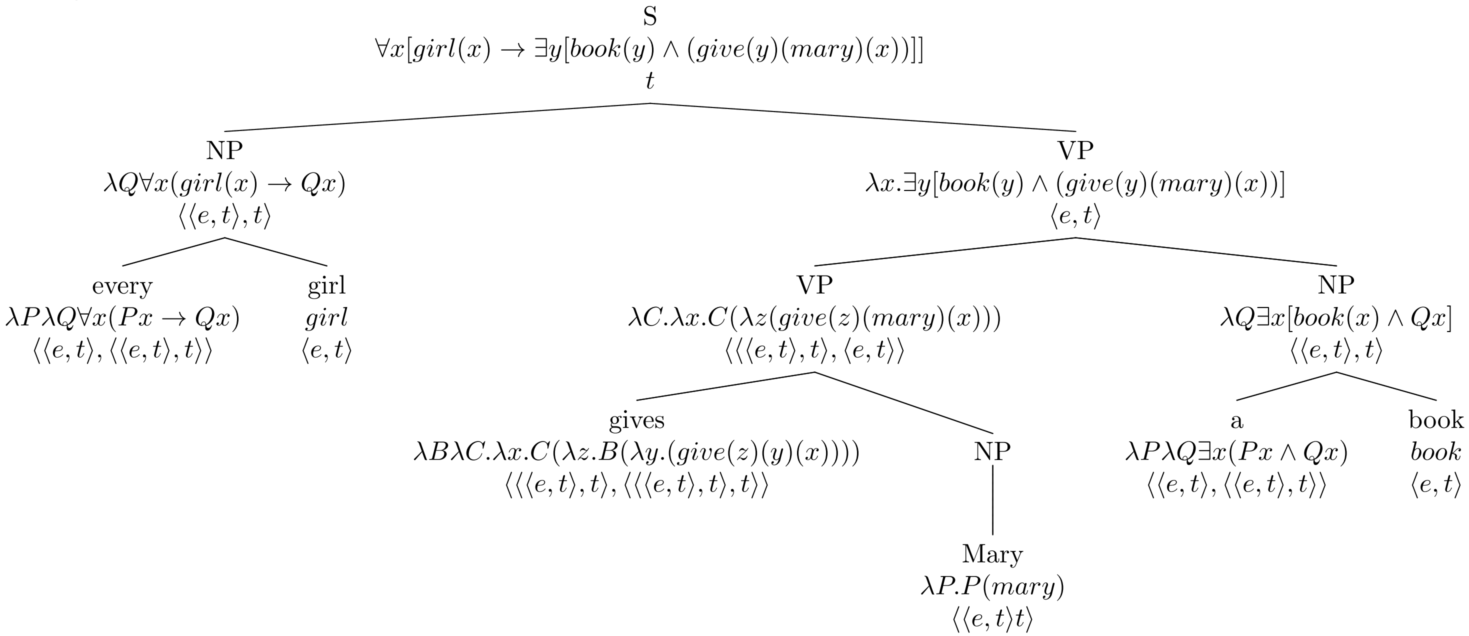
(a)

lexicon:
every $\lambda P \lambda Q \forall x[Px \wedge Qx]$
girl $girl$
gives (reading one) $\lambda B.\lambda C.\lambda AC(\lambda x A(\lambda z(B(\lambda y.(give(x)(y)(z))))))$
gives (reading two) $\lambda B \lambda C.\lambda x.C(\lambda z.B(\lambda y.(give(z)(y)(x))))$
mary $\lambda P.P(mary)$
a $\lambda P.\lambda Q.\exists x[Px \wedge Qx]$
book $book$
where A, B, C are of type $\langle\langle e, t\rangle, t\rangle$
Instead of the complicated rule system of EMG I use only two rules: when the left child can be applied to the right, do so, and vice versa.

Reading one:


S
$\exists x[book(x) \wedge \forall y[girl(y) \rightarrow (give(x)(mary)(y))]]$
t
NP
$\lambda Q \forall x(girl(x) \rightarrow Qx)$
$\langle\langle e,t\rangle,t\rangle$
VP
$\lambda A.\exists x[book(x) \wedge A(\lambda z(give(x)(mary)(z)))]$
$\langle\langle\langle e,t\rangle,t\rangle,t\rangle$
every
$\lambda P \lambda Q \forall x(Px \rightarrow Qx)$
$\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
girl
$girl$
$\langle e,t\rangle$
VP
$\lambda C \lambda A.C(\lambda x A(\lambda z(give(x)(mary)(z))))$
$\langle\langle\langle e,t\rangle,t\rangle,\langle\langle\langle e,t\rangle,t\rangle,t\rangle\rangle$
NP
$\lambda Q \exists x[book(x) \wedge Qx]$
$\langle\langle e,t\rangle,t\rangle$
gives
$\lambda B \lambda C \lambda A.C(\lambda x A(\lambda z(B(\lambda y.(give(x)(y)(z))))))$
$\langle\langle\langle e,t\rangle,t\rangle,\langle\langle\langle e,t\rangle,t\rangle,\langle\langle\langle e,t\rangle,t\rangle,t\rangle\rangle\rangle$
NP
Mary
$\lambda P.P(mary)$
$\langle\langle e,t\rangle t\rangle$
a
$\lambda P \lambda Q \exists x(Px \wedge Qx)$
$\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
book
$book$
$\langle e,t\rangle$

Reading two:


S
$\forall x[girl(x) \rightarrow \exists y[book(y) \wedge (give(y)(mary)(x))]]$
t
NP
$\lambda Q \forall x(girl(x) \rightarrow Qx)$
$\langle\langle e,t\rangle,t\rangle$
VP
$\lambda x.\exists y[book(y) \wedge (give(y)(mary)(x))]$
$\langle e,t\rangle$
every
$\lambda P \lambda Q \forall x(Px \rightarrow Qx)$
$\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
girl
$girl$
$\langle e,t\rangle$
VP
$\lambda C.\lambda x.C(\lambda z(give(z)(mary)(x)))$
$\langle\langle\langle e,t\rangle,t\rangle,\langle e,t\rangle\rangle$
NP
$\lambda Q \exists x[book(x) \wedge Qx]$
$\langle\langle e,t\rangle,t\rangle$
gives
$\lambda B \lambda C.\lambda x.C(\lambda z.B(\lambda y.(give(z)(y)(x))))$
$\langle\langle\langle e,t\rangle,t\rangle,\langle\langle\langle e,t\rangle,t\rangle,t\rangle\rangle$
NP
Mary
$\lambda P.P(mary)$
$\langle\langle e,t\rangle t\rangle$
a
$\lambda P \lambda Q \exists x(Px \wedge Qx)$
$\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
book
$book$
$\langle e,t\rangle$

(b) More lexicon:

John $\lambda P.P(john)$

hits $\lambda A.\lambda x.A(\lambda y.(hits(y)(x)))$

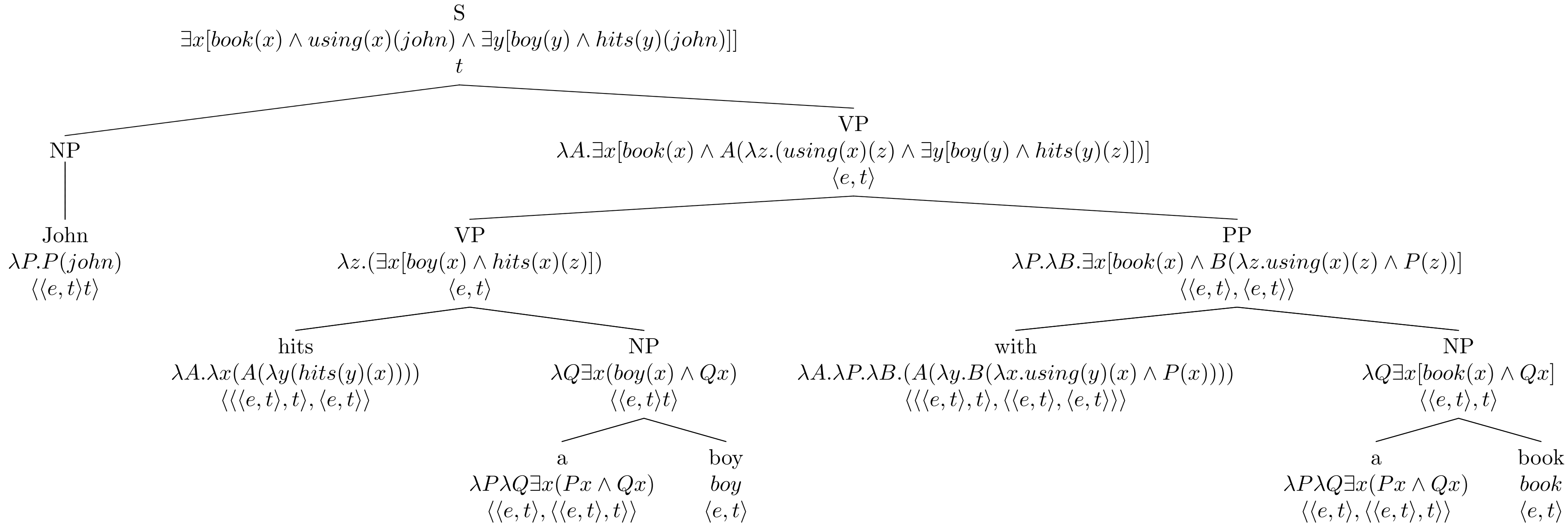
with (reading one) $\lambda A.\lambda B.\lambda P.A(\lambda y.B(\lambda x.using(y)(P(x))))$

with (reading two) $\lambda A.\lambda B.\lambda P.A(\lambda y.B(\lambda x.has(x)(y) \wedge P(y)))$

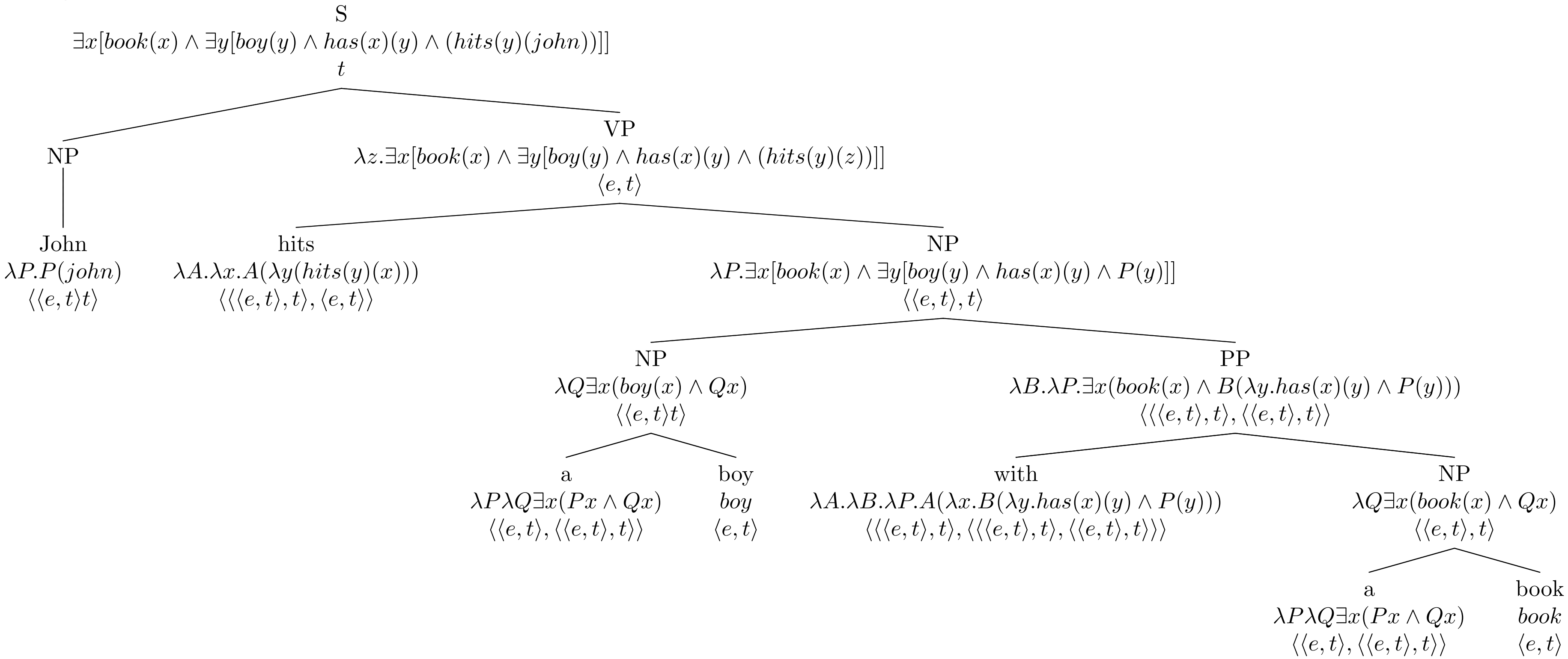
Reading one, first, failed attempt, where *using* modifies the *hits* predicate:

- S — $\exists x[book(x) \wedge \exists y[boy(y) \wedge using(x)(hits(y)(john))]]$ — t
  - NP
    - John — $\lambda P.P(john)$ — $\langle\langle e,t\rangle t\rangle$
  - VP — $\lambda z \exists x[book(x) \wedge \exists y[boy(y) \wedge using(x)(hits(y)(z))]]$ — $\langle e,t\rangle$
    - VP — $\lambda y.(\exists x[boy(x) \wedge hits(y)(x)])$ — $\langle e,t\rangle$
      - hits — $\lambda A.\lambda x(A(\lambda y(hits(y)(x))))$ — $\langle\langle\langle e,t\rangle,t\rangle,\langle e,t\rangle\rangle$
      - NP — $\lambda Q \exists x(boy(x) \wedge Qx)$ — $\langle\langle e,t\rangle t\rangle$
        - a — $\lambda P \lambda Q \exists x(Px \wedge Qx)$ — $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
        - boy — *boy* — $\langle e,t\rangle$
    - PP — $\lambda B.\exists x[book(x) \wedge B(\lambda P.\lambda z.using(x)(P(z)))]$ — $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$
      - with — $\lambda A.\lambda B.(A(\lambda y.B(\lambda P.\lambda x.using(y)(P(x)))))$ — $\langle\langle\langle e,t\rangle,t\rangle,\langle\langle e,t\rangle,\langle e,t\rangle\rangle\rangle$
      - NP — $\lambda Q \exists x[book(x) \wedge Qx]$ — $\langle\langle e,t\rangle,t\rangle$
        - a — $\lambda P \lambda Q \exists x(Px \wedge Qx)$ — $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
        - book — *book* — $\langle e,t\rangle$

Reading one, second attempt, where *using* is a separate predicate:
with (reading one) : $\lambda A.\lambda P.\lambda B.A(\lambda y.B(\lambda x.using(y)(x) \wedge (P(x))))$



Reading two:


S
∃x[book(x) ∧ ∃y[boy(y) ∧ has(x)(y) ∧ (hits(y)(john))]]
t
NP
John
λP.P(john)
⟨⟨e, t⟩t⟩
VP
λz.∃x[book(x) ∧ ∃y[boy(y) ∧ has(x)(y) ∧ (hits(y)(z))]]
⟨e, t⟩
hits
λA.λx.A(λy(hits(y)(x)))
⟨⟨⟨e, t⟩, t⟩, ⟨e, t⟩⟩
NP
λP.∃x[book(x) ∧ ∃y[boy(y) ∧ has(x)(y) ∧ P(y)]]
⟨⟨e, t⟩, t⟩
NP
λQ∃x(boy(x) ∧ Qx)
⟨⟨e, t⟩t⟩
a
λPλQ∃x(Px ∧ Qx)
⟨⟨e, t⟩, ⟨⟨e, t⟩, t⟩⟩
boy
boy
⟨e, t⟩
PP
λB.λP.∃x(book(x) ∧ B(λy.has(x)(y) ∧ P(y)))
⟨⟨⟨e, t⟩, t⟩, ⟨⟨e, t⟩, t⟩⟩
with
λA.λB.λP.A(λx.B(λy.has(x)(y) ∧ P(y)))
⟨⟨⟨e, t⟩, t⟩, ⟨⟨⟨e, t⟩, t⟩, ⟨⟨e, t⟩, t⟩⟩⟩
NP
λQ∃x(book(x) ∧ Qx)
⟨⟨e, t⟩, t⟩
a
λPλQ∃x(Px ∧ Qx)
⟨⟨e, t⟩, ⟨⟨e, t⟩, t⟩⟩
book
book
⟨e, t⟩

Exercise 5

More lexicon:

smokes : *smokes*
and : $\lambda P \lambda A \lambda y.A(\lambda x(P(x) \wedge P(y)))$
Bill : $\lambda P.P(bill)$
does : $\lambda A.A$
too : $\lambda D.D$
where D is of type $\langle\langle\langle e,t\rangle,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$
Apologies for the unorthodox phrase structure, but this is what my semantic intuitions told me.