Meaning, Reference & Modality. Assignment 1

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Exercise 1

Equality is neither a relation between objects, nor a relation between names. If it were a relation between objects it would not be informative, there would be no difference between a=a and a=b, whereas these are very different in practice: the first is analytic, the second potentially has cognitive value.

If equality were a relation between names then it would only be about language. However, the discovery that the morning star is the evening star was a groundbreaking astronomical discovery, requiring diligent work and not just re-arragenments in language. The reference of an equality statement is its truth value, and the sense is its mode of presentation. This explains why a=a and a=b can both be true, having the same reference, whereas they differ in cognitive value, because of their senses.

Exercise 2

No, the principle of extensionality does not fail in Frege's theory, because of the way the distinction between sense and reference is employed. In intensional contexts there is 'indirect reference': the reference of an expression is its sense. This means that expressions can be freely substituted, given equal senses.

Exercise 3

The first objection is that it would be circular or incorrect. Suppose "possibly phi" means that phi is consistent. Here consistent means that it is possible to find some extension that makes phi true, so in defining 'consistent' we are back at 'possible' again. A more sophisticated versions says that the denial of phi is not among its theorems, but this runs into problems because of Godel's incompleteness theorem:

"either there are falsehoods among its theorems or there is some falsehood of arithmetic whose denial is not among its theorems"

The second objection is that possibilities are not just linguistic entities. We might take modal operators as primitive and view "possible worlds" as representations using eg., sets of consistent sentences. However, because Lewis believes that possible worlds are of the same kinds as the actual world, this would lead us to conclude that our actual world is a set of sentences, which is absurd. Lewis believes that possible worlds should be taken as primitive, so that modal operators can simply be viewed as quantifiers ranging over these

worlds.

Lewis' ideas imply that it is not possible to know much about possibilities, except that they are of the same kind as our world. Because Lewis really believes in possibilities he can not say more about them than that they exist, as opposed to someone who only pretends to believe in them, who can consequently make up whatever he likes. The best we can do is use our intuitions about the way things might have been.

Exercise 4

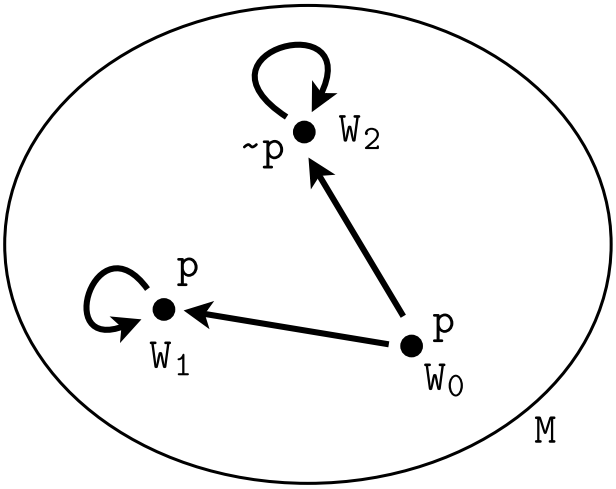
The first objection is that Lewis conflates the state of the world with the world itself. This allows Lewis to go from affirming that there are 'ways in which things might have been' to the existence of concrete possible worlds. Stalnaker concludes that while possible worlds exist, they do not have to have the same status as our world. Our world actually exists and possible worlds are abstract properties about our actual world.

The second objection concerns the indexical analysis of actuality. If actuality would merely be an indexical depending on the context, ie., the world, from which it is uttered, then all worlds are actual in their own right without any way to distinguish them. This is because there is no absolute, world-neutral standpoint to make such an observation. One is always looking from within some world. Stalnaker points out that Lewis' theory that actuality is an indexical relation is only a semantical thesis, from which no metaphysical conclusions should be drawn.

Lewis' two demands for a metaphysical theory are that it is 1) systematic and 2) agrees with prephilosophical intuitions. Stalnaker's theory is certainly systematic, and I believe it also better agrees with prephilosophical intutions, because it is unintiuitive to believe that possible worlds have a concrete existence even though they are completely inaccessible to us.

Exercise 5

World 1 makes p possible, world 2 makes ~p possible. Because world 1 and 2 are only connected to themselves, in each of them either p or ~p is necessariliy true. Hence from world 0 the formula holds.



Exercise 6

The formula <>(p -> q) -> (<>p -> <>q) characterizes the property that for each world there is at most one connection. Formally this can be described as:

for all w, v, u in W: if wRv and wRu, then v=u.

This property is also called "partially functional". An easier way to characterize frames with this property is:

<>p -> []p

Outline of proof:

Suppose R is a partially functional relation on W in F for some w in W:
- suppose p -> q is not possible from w, then the conditional is true.
- alternatively, suppose p -> q is possible from w, then we have wRw' where p -> q holds in w'
- the three cases in which p -> q is possible are: {p, q}, {~p, q}, {~p, ~q}. By the definition of implication all these valuations succeed in making '<>p -> <>q' true.

Suppose R is not a partially functional relation on W in F for some w in W:
- then there are w, v, u such that wRv and wRu but v is not u.
- suppose p -> q is possible from w, say in v, where both p and q are false.
- suppose in world u, p is true yet q is false.
- in this case the first part of the antecedent succeeds because of world v, yet the second fails because of world u.