

Meaning, Reference and Modality 11

Multidimensional Meaning

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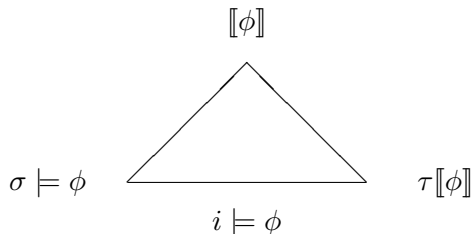
Outline

- ① Background
 - ② Pronominalization
 - ③ Presupposition
 - ④ Quantification
 - ⑤ Scope Islands
 - ⑥ Peirce Puzzle ???
- 2004, “The pragmatic dimension of indefinites,” *Research on Language and Computation* 2, pp. 365–399.
 - 2008, “A multi-dimensional treatment of quantification in extraordinary English,” *Linguistics and Philosophy* 31, pp. 101–127.

Semantics and Pragmatics

- The question what well-formed expressions mean. (Semantics)
 - ▶ Tarski, Montague, Davidson,
- The question what people do with them. (Pragmatics)
 - ▶ Strawson, Grice, Kripke, Stalnaker,
- Can these questions be dealt with separately?
 - ▶ Maybe better not. (Dynamic Semantics)
- Should a distinction be maintained?
 - ▶ Regarding pronouns, presupposition, reference, and quantification?

Core Notions in a Scheme



- $i \models \phi$: satisfaction ('logic');
- $\llbracket \phi \rrbracket = \{i \mid i \models \phi\}$: meaning ('philosophy');
- $\sigma \models \phi$ iff $\sigma \subseteq \llbracket \phi \rrbracket$: support ('pragmatics');
- $\tau \llbracket \phi \rrbracket = \tau \cap \llbracket \phi \rrbracket$: update ('pragmatics').
- This works for indicatives and interrogatives.

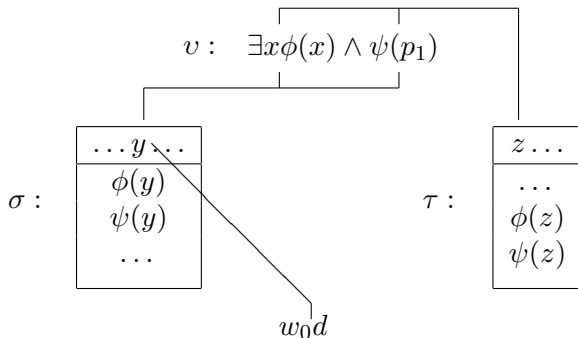
Some Desirable Features

- An independent compositional definition of $\llbracket \bullet \rrbracket$, \models , and $\tau[\llbracket \bullet \rrbracket]$.
- If $\sigma \models \phi$ then $(\sigma \cap \tau) \subseteq \tau[\llbracket \phi \rrbracket]$.
- This sets the stage for extending the picture with pronouns, presuppositions, first and second order quantifiers.

Discourse Facts

- Relevance of the (discourse) fact that an (in)definite term has been used (with referential intentions) shows:
 - ▶ the need of a level of discourse representation (Kamp);
 - ▶ of a richer notion of meaning (Groenendijk and Stokhof);
 - ▶ the interaction between semantic and pragmatic information (Stalnaker).
- The pragmatic assumption that there exists a true answer to the question which individual a term stands for (in some causal intentional chain; cf., e.g., Chastain 1975, Donnellan 1978, Kamp 1981, Slater 1986, Zimmermann 1998, Stalnaker 1998).

Representing Supported Updates



- The piece-wise and compositional presentation of aggregated information.

Constructive Interpretation

Dynamic Predicate Logic

- Destructive assignment procedure.

Sequence Semantic Solution (Kees Vermeulen)

- Stack new values of a variable x on old values:

$$\triangleright g(x) = d_1 d_2 \dots$$

- Stack values of all variables on one variable:

- ▶ A farmer owns a donkey. He beats it.

$$\exists x(Fx \wedge \exists x(Dx \wedge O \downarrow x \uparrow x)) \wedge B \downarrow x \uparrow x$$

- Stack values on no variables (Jan van Eijck, Me):

- ▶ A farmer owns a donkey. He beats it.

$$\exists(F_ \wedge \exists(D_ \wedge O \downarrow _ \uparrow _)) \wedge B \downarrow _ \uparrow _$$

Cleaning Up

Coreference and Modality (GSV)

- information states are sets of possibilities, triples $i = \langle r, g, w \rangle$

Cleaning Up (PD)

- possibilities are sequences of individuals plus a world
- information states are sets of worlds, interpretation is relative to a sequence of individual concepts

Cleaning Up DMPL Information States

- | | | | | | |
|----------|----------|----------|----------|---------|----------|
| | x | $-$ | y | \dots | z |
| $n:$ | 0 | 1 | 2 | \dots | $n-1$ |
| w_1 | a | a' | a'' | \dots | a''' |
| w_2 | b | b' | b'' | \dots | b''' |
| \vdots | \vdots | \vdots | \vdots | \dots | \vdots |

- | | | | | | |
|----------|----------|----------|----------|---------|----------|
| $n:$ | 0 | 1 | 2 | \dots | $n-1$ |
| w_1 | a | a' | a'' | \dots | a''' |
| w_2 | b | b' | b'' | \dots | b''' |
| \vdots | \vdots | \vdots | \vdots | \dots | \vdots |

»

w_1	a	a'	a''	\dots	a'''
w_2	b	b'	b''	\dots	b'''
\vdots	\vdots	\vdots	\vdots	\dots	\vdots

Frege's Donkey Imperative

- “Wenn jemand heute dasselbe sagen will, was er gestern das Wort “heute” gebrauchend ausgedrückt hat, so wird er dieses Wort durch “gestern” ersetzen.”
- The ‘character’ of the conjunction of (i) an utterance of “Es regnet heute” on Tuesday and (ii) that of “Es regnet heute nicht” on Wednesday is given, on Wednesday, by “Es regnete gestern und es regnet heute nicht.”
 - ▶ $R(h)_{tue} \wedge \neg R(h)_{wed} \Leftrightarrow (R(y) \wedge \neg R(h))_{wed}$
- Website philosophy:
 - ▶ a practical bibliographical order has the historical order reversed.
- Pronoun p_1 picks up the referent introduced last, p_2 picks up the referent introduced one but last,

PLA, A Minimal Satisfaction Semantics

- L , a language of first order predicate logic with pronouns p_i .
- $M, g, e \models \phi$, with M a first order model, g a variable assignment; e a sequence of possible witnesses.
- $[c]_{M,g,e} = M(c)$ $[x]_{M,g,e} = g(x)$ $[p_i]_{M,g,e} = e_i$
- $M, g, e \models Rt_1 \dots t_m$ iff $\langle [t_1]_{M,g,e}, \dots, [t_m]_{M,g,e} \rangle \in M(R)$
 $M, g, e \models \neg\phi$ iff $M, g, ce \models \phi$ for no $c \in E^{n(\phi)}$
 $M, g, dce \models \exists x\phi$ iff $M, g[x/d], ce \models \phi$
 $M, g, ace \models \phi \wedge \psi$ iff $M, g, ce \models \phi$ and $M, g, ace \models \psi$

One Basic Example

(1) A farmer owns a donkey. He beats it with a stick.

(1') $\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \wedge \exists z(Sz \wedge Bp_1p_2z)$

$M, g, sfde \models (1')$ iff

$\gg M, g, fde \models \exists x(Fx \wedge \exists y(Dy \wedge Oxy))$ and
 $M, g, sfde \models \exists z(Sz \wedge Bp_1p_2z)$ iff

$\blacktriangleright M, g[x/f][y/d], e \models (Fx \wedge Dy \wedge Oxy)$ and
 $M, g[z/s], fde \models (Sz \wedge Bp_1p_2z)$ iff

$\star \langle f, d \rangle \in ((M(F) \times M(D)) \cap M(O))$ and
 $f \in M(S)$ and $\langle f, d, s \rangle \in M(B)$

Egli's Theorems Renewed

- (2) A diver found a pearl but she lost it again.
A diver lost a pearl she just found.

$$\bullet (\exists x(Dx \wedge \exists y(Py \wedge Fxy)) \wedge Lp_1p_2) \Leftrightarrow \exists x(Dx \wedge \exists y((Py \wedge Fxy) \wedge Lxy))$$

- (3) If a farmer owns a donkey he beats it.
Every farmer beats every donkey he owns.

$$\bullet (\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow Bp_1p_2) \Leftrightarrow \forall x(Fx \rightarrow \forall y((Dy \wedge Oxy) \rightarrow Bxy))$$

Some Facts about PLA

- PLA has a classical satisfaction relation, covering intersentential anaphoric relationships and dynamic entailment, as in *DPL*.
- Unlike *D(M)PL*, *PLA*:
 - ▶ employs the usual notions of scope and binding,
 - ▶ preserves α -conversion,
 - ▶ does not raise any issue of downdating.
- The dynamics resides in the *use* of witnesses, and
- a dynamic, that is, *pragmatic*, notion of conjunction.
 - ▶ (But not necessarily inflexibly.)

Presupposition, A Short History (part zero)

- Aristotle
 - Pedro Julião (Aka. Petrus Hispanus, Pope John XXI)
 - Gottlob Frege, Bertrand Russell, Peter Strawson
- ① Kepler did not die / died in misery.
 - ② Odysseus wurde tief schlafend in Ithaka ans Land gesetzt.
 - ③ The present king of France is (not) wise.
 - ④ Pegasus does not exist.
 - ⑤ The table is covered with books.

Presupposition, Some Examples (part one)

- ① John found the **lost pack of cigarettes**.
- ② Ron realized **what a good boy John was**.
- ③ All **kids** thought they were alone.
- ④ [Harry]_F **smoked**.
- ⑤ It was Bert **who lit the cigarette**.
- ⑥ Marry **smoked**, too.
- ⑦ No, only [**boys**]_F **smoked**.
- ⑧ Amelia had stopped **smoking**.

Presupposition Theory (part two)

- A linguistic hype in the 1970-ies, a theory!
 - ▶ Projection, cancellation, accommodation;
 - ▶ Plugs, holes, filters.
 - ▶ Karttunen, Peters, Gazdar, ...
- A glorious revival in the 1980-ies and 90-ies, a logic!
- Satisfaction theory (Stalnaker, Heim, Beaver);
- binding theory (van der Sandt, Geurts, Kamp);
- recent developments (Asher, Schlenker, Chemla).

Satisfaction Theory

- Stalnaker: a (pragmatic) presupposition is one a speaker assumes in a given context and assumes his addressees to be assuming, too.
- Notation: a formula ϕ_χ presupposes that χ and asserts ϕ .
- In a dynamic (pragmatic) semantics,
 - ▶ $\sigma[\![\phi_\chi]\!] = \sigma[\![\phi]\!]$ provided that
 - ★ $\sigma \models \chi$; undefined otherwise.
- Cool predictions:
 - ▶ $(\phi \wedge \psi_\phi)$ and $(\phi \rightarrow \psi_\phi)$ have no presuppositions.
 - ▶ John has children, and his children are bald.
 - ▶ If John has children, his children are bald.

Satisfaction Theoretic Problems

- Not so cool predictions:
 - ▶ $(\phi \rightarrow \psi_\chi)$ just presupposes $(\phi \rightarrow \chi)$.

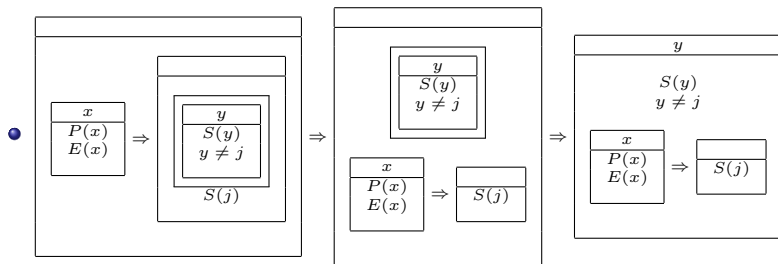
Gazdar, Van der Sandt and Geurts' "proviso problem".
- If the package is empty, then John has smoked, too.
 - ▶ Presupposes that somebody smoked, the package being empty or not.
- If I torture him, Bill will regret having laughed at me.
 - ▶ Presupposes that Bill laughed at me, if I tortured him or not.
- If all boys failed the exam, it wasn't only Fred who failed.
 - ▶ Presupposes that Fred failed, whether or not all other boys failed as well.

Accommodation and Binding

- “(...) presuppositional expressions are claimed to be anaphoric expressions which have internal structure and semantic content.” (van der Sandt 1992)
- “If their capacity to accommodate is taken into account they can be treated by basically the same mechanism which handles the resolution of pronouns.” (van der Sandt 1992)
- Presupposition is a species of anaphora (van der Sandt), or better, pronouns are presuppositional (Geurts).
- Discourse *Representation* Theory (Kamp, van der Sandt, Geurts):
 - presuppositions and pronouns are handled well;
 - but they are not interpreted;
 - resolution boils down to elimination.

Accommodation and Binding in DRT

- If the package is empty, John has smoked, too.



- This is the favoured one of three resolutions.

Questions

Good Questions

- What is the meaning of a *DRS*?
- What is the interpretation of a presupposition?

Good Answers

- The meaning of a DRS resides in its satisfaction conditions.
- Presuppositions are informational objects in their own right.
- Like pronouns, they have to be handled with care.

Old Implementation

- Two-dimensional interpretation (Karttunen and Peters, 1979).

Two-Dimensional Interpretation

- Every expression has a presupposition (or: ‘implicature’) and an assertion (or: ‘extension’) in its proper denotation type.
- “Manage to raise my interest”
 - denotation type: property of individuals.
- It ‘presupposes’ the set of individuals who try to raise my interest;
- and it ‘asserts’ the set of individuals who succeed in doing so.
- That’s as simple as it is. Presupposition and assertion are possibly independent creatures.
- “Jones is raking the leaves over there.” Presupposition may fail, while the assertion succeeds!

A Management Problem (K&P 1979)

- Cross-dimensional ‘binding’.
 - (1) Somebody managed to succeed George V.
 - (1) presupposes ?Somebody tried hard to succeed George V
 - (1) asserts ?Somebody eventually succeeded
- Cross-dimensional ‘resolution’.
 - (1) presupposes that someone tried hard to succeed George V
 - (1) asserts that *he* (that person) eventually succeeded
- But this *already expected* solution is something we can handle (Kamp, Heim, van der Sandt, dynamic semantics)

Presupposition in PLA (Toy Model)

- $M, g, e \models^p MAN(t, s)$ iff $\langle [t]_{M,g,e}, [s]_{M,g,e} \rangle \in M(TRY)$
 $M, g, e \models^a MAN(t, s)$ iff $\langle [t]_{M,g,e}, [s]_{M,g,e} \rangle \in M(SUC)$
- $M, g, e \models^p \neg\phi$ iff $\exists c: M, g, ce \models^p \phi$
 $M, g, e \models^a \neg\phi$ iff $\neg\exists c: M, g, ce \models^a \phi$
- $M, g, de \models^p \exists x\phi$ iff $M, g[x/d], e \models^p \phi$
 $M, g, de \models^a \exists x\phi$ iff $M, g[x/d], e \models^a \phi$
- $M, g, ace \models^p (\phi \wedge \psi)$ iff $M, g, ce \models^p \phi$ and $M, g, ace \models^p \psi$
 $M, g, ace \models^a (\phi \wedge \psi)$ iff $M, g, ce \models^a \phi$ and $M, g, ace \models^a \psi$
- $\models = (\models^p \cap \models^a)$

Solving a Management Problem

(1) Someone managed to succeed George V on the throne.

- $M, g, de \models^p \exists xMANxs$ iff d tried s
 $M, g, de \models^a \exists xMANxs$ iff d succeeded with s
- utterly simple indeed

(2) Noone managed to succeed George V.

- $M, g, e \models^p \neg \exists xMANxs$ iff some d tried s
 $M, g, e \models^a \neg \exists xMANxs$ iff no d succeeded with s

Three-Dimensional Interpretation

- Going fully compositional *and* add another dimension:
 - ▶ indefinites contribute (not assert) discourse referents,
 - ▶ but they are not ‘presuppositional’.

- (2) Yesterday some student came by to hear my views on free logic.
 - (3) No, nobody came to know anything from you.
 - (3′) No, she really wanted to see your new haircut.
 - (3′′) ??No, she is not a student any longer.

- Distinguish assertoric, presuppositional and other aspects of interpretation (Karttunen and Peters).

- (4) Pedro hired a donkey.
 - ▶ Presupposes a guy p named ‘Pedro’,
 - ▶ introduces a donkey d ,
 - ▶ asserts that p hired d .

The Generalized Quantifier Interpretation of Terms

- A uniform treatment of (all?) noun-phrases.
- “John” is interpreted as the set of John’s properties.
- $\llbracket \text{JOHN} \rrbracket = \{Q \mid Q(j)\}$, with j being John.
 $Q \in \llbracket \text{JOHN} \rrbracket$ iff Q is a property of John.
- $\llbracket \text{ALL_POPE} \rrbracket = \{Q \mid P \subseteq Q\}$, with P the set of all popes.
 $Q \in \llbracket \text{ALL_POPE} \rrbracket$ iff Q is a property of all popes.
- $\llbracket \text{SOME_PIG} \rrbracket = \{Q \mid P \cap Q \neq \emptyset\}$, with P the set of pigs.
 $Q \in \llbracket \text{SOME_PIG} \rrbracket$ iff Q is a property of some pig.
- ...

PLA/PTPQ Interpretation of Terms

- $$\llbracket \text{MARY} \rrbracket_{M,g,de}^p = \{Q \mid d = [m]_{M,g,e} \ \& \ d \in Q\}$$

$$\llbracket \text{MARY} \rrbracket_{M,g,de}^c = \{Q \mid d \in Q\}$$

$$\llbracket \text{MARY} \rrbracket_{M,g,de}^a = \{Q \mid d \in Q\}$$
- $$\llbracket \text{HE}_i \rrbracket_{M,g,de}^p = \{Q \mid d = e_i \ \& \ d \in Q\}$$

$$\llbracket \text{HE}_i \rrbracket_{M,g,de}^c = \{Q \mid d \in Q\}$$

$$\llbracket \text{HE}_i \rrbracket_{M,g,de}^a = \{Q \mid d \in Q\}$$
- $$\llbracket \text{SOME}(\pi) \rrbracket_{M,g,dce}^p = \{Q \mid d \in \llbracket \pi \rrbracket_{M,g,ce}^p \ \& \ d \in Q\}$$

$$\llbracket \text{SOME}(\pi) \rrbracket_{M,g,dce}^c = \{Q \mid d \in \llbracket \pi \rrbracket_{M,g,ce}^{c\&a} \ \& \ d \in Q\}$$

$$\llbracket \text{SOME}(\pi) \rrbracket_{M,g,dce}^a = \{Q \mid d \in Q\}$$
- $$M, g, dace \models^x T(\rho) \text{ iff } \llbracket \rho \rrbracket_{M,g,ace}^x \in \llbracket T \rrbracket_{M,g,dce}^x$$

Two Easy Examples

- (1) Mary teased some dude.

$$MARY(\lambda x \text{ SOME}(DUDE)(\lambda y \text{ TEASE}xy))$$

- ▶ $M, g, mde \models^p (1)$ iff $m = [m]_{M,g,e}$
- $M, g, mde \models^c (1)$ iff $d \in M(DUDE)$
- $M, g, mde \models^a (1)$ iff $\langle m, d \rangle \in M(TEASE)$

- (2) She₁ was annoyed by a song he₂ sang.

$$SHE_1(\lambda x \text{ SOME}(SONG-SUNG-BY-HE_2)(\lambda y \text{ ANNOY}yx))$$

- ▶ $M, g, nbsmde \models^p (2)$ iff $n = m$ and $b = d$
- $M, g, nbsmde \models^c (2)$ iff $\langle b, s \rangle \in M(SUNG-SONG)$
- $M, g, nbsmde \models^a (2)$ iff $\langle s, n \rangle \in M(ANNOY)$

Interpretation of Quantifiers

- $\llbracket \lambda z \phi \rrbracket_{M,g,fh} = \{d \mid M, g[z/d], (fh)(d) \models \phi\}$
 $\llbracket !\pi \rrbracket_{M,g,fh} = \{d \mid \exists c: d \in \llbracket \pi \rrbracket_{M,g,ch} \ \& \ f = \{\langle d, c \rangle \mid d \in \llbracket \pi \rrbracket_{M,g,ch}\}\}$
- $\llbracket DET(\pi) \rrbracket_{M,g,Dfh}^p = \{Q \mid \emptyset \neq D = \llbracket !\pi \rrbracket_{M,g,fh} \subseteq Q \ \& \ FC(f)\}$
 $\llbracket DET(\pi) \rrbracket_{M,g,Dfh}^c = \{Q \mid D \subseteq Q\}$
 $\llbracket DET(\pi) \rrbracket_{M,g,Dfh}^a = \{Q \mid \langle D, Q \rangle \in [DET]\}$
- $FC(f)$ requires f to be a function.
- Fairly standard again.
- Caveat: different types of witness sets may be needed.

Some More Involved Example

- Most man who sent a present to Curt sent a different₂ present to Amelia.

$$\begin{aligned}
 &MOST(\lambda x Mx \wedge CURT(\lambda y SOM(P)(\lambda z Sxyz))) \\
 &\quad (\lambda x AMEL(\lambda y SOMDIFF_2(P)(\lambda z Sxyz)))
 \end{aligned}$$

- » Presupposes c is Curt, a is Amelia,
 D is the set of men d who sent a present, viz. $p(d)$, to Curt;
- » contributes q : $q(d)$ is a present different from $p(d)$;
- » asserts $MOST d \in D$ sent $q(d)$ to a .

In DRT Format

- Most who sent a present to Curt sent a different₂ one to Amelia.

c, a, D, p	q	
$c = \text{Curt}, a = \text{Amelia}$ $D = \lambda d \text{ prest}(p(d)) \ \&$ $\text{sent}(d, c, p(d))$	$\text{prest}(q(d))$ $q(d) \neq p(d)$	$\text{MOST}(D)$ $(\lambda d \text{ sent}(d, a, q(d)))$

Further Applications

- (2) Every boy has a gun, but hardly any boy ever uses it.
 - (3) Sue uses it to chase foxes by the way.
- Indeed, if it is a boy named Sue.

A Flexible Treatment of Presupposition Resolution

(1) Everyone moved to Stuttgart because a woman lived there.

- ... because Stuttgart is not 100% male?
 - ▶ quite unlikely, why not Amsterdam? Nijmegen?
- ... because Dorit Abusch lived there?
 - ▶ bad reason: Dorit left.

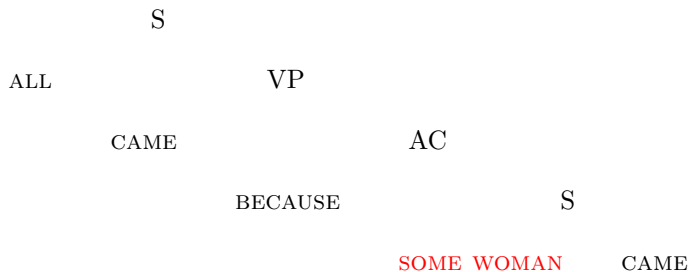
» ... because everyone's beloved one lived there.

- ▶ $f: f(d)$ was the reason for d to move (for $d \in D$).

Island Constraints

- Quantified expressions cannot escape scope islands (...):
 - (1) If many vegetarians come to the party, Max will have a problem.
 - (2) Max had a big problem, because many veg.s came to the party.
- (...) whereas indefinite expressions can, or so they say.
 - (3) If we invite a certain philosopher Max will be offended, but do you remember who? (Reinhart)
 - (4) Max did not consider the possibility that some politician is corrupt. Who, by the way? (Kratzer)
- Why do indefinites seem to do what syntax forbids them to do?
- Presuppositions can 'swim'! (Geurts)

Dorit Abusch Again



Dorit in Detail

(1) Everybody came because a woman came.

$ALL(\lambda x \text{BEC}(SOM(WO)(CA), CAx))$

- $M, g, pqace \models^p \text{BEC}(\psi, \phi)$ iff $M, g, ce \models \phi$ and $M, g, ace \models \psi$
 $M, g, pqace \models^c \text{BEC}(\psi, \phi)$ iff $p = \llbracket \phi \rrbracket_{M,g,ce}^a$ and $q = \llbracket \psi \rrbracket_{M,g,ace}^a$
 $M, g, pqace \models^a \text{BEC}(\psi, \phi)$ iff $\langle q, p \rangle \in M(\text{BEC})$
- The contribution of $SOM(WO)(CA)$ can stay where it is delivered, or it can be transparent (specific or functional).
- The functional reading in a representational format:

D, f	p, q	
<ul style="list-style-type: none"> $\forall d \in D:$ $CA(d)$ $WOCA(f(d))$ 	$\forall d \in D:$ $p(d) = CA(d)$ $q(d) = WOCA(f(d))$	$\forall d \in D:$ $\text{BEC}(q(d), p(d))$

Island Constraints Without Violations

- No violations; it is the pragmatic contribution of an indefinite, not its assertoric force, which is insensitive to the constraint // “presuppositions can swim”.
- The assertoric contribution is a witness $f(d)$ figuring right there at the logical level where the indefinite occurs.
- That there are pragmatic effects associated with the use of indefinites is orthogonal to their semantics.
- Thus, indefinites behave like other noun phrases:
 - ▶ proper names, pronouns, definites, . . . , and
 - ▶ **generalized quantifiers as well.**

(22) If a boy goes to a party because he thinks **most girls in his class** come too, he is disappointed, of course, when **they** don't show up.

So Far so Good? Not Really!

- (1) Everyone visited a student in Stuttgart because a professor told him to do so.
 - Two of at least four interpretations:
 1. everyone visited an arbitrary student because a specific professor told him to do so;
 2. everyone visited a specific student because an arbitrary professor told him to visit that student.
 - Not a likely interpretation is one according to which some subjects satisfy one of these conditions and all the others the other.
 - The questions: “which student” and “which professor” would become awkward; the answer would have to be: “that depends on whom we are evaluating”.
 - We can exclude this by indicating the pragmatic mode of composition on the level of logical form.

An Example After Philippe Schlenker

- (2) If each student finishes a paper on Chomsky in time and improves in two subjects, then noone will fail the exam.
- This may concern an arbitrary paper on Chomsky, a very specific one, and one specific to each student; and it may concern two arbitrary subjects, two specific ones, or two ones specific to each student (this is already nine interpretations).
 - But I can see none where **for some** students there is required to be **an arbitrary paper** but **two specific subjects**, and **for others a specific paper** and **two just arbitrary subjects**.
 - Pragmatic mode of composition must be visible at the level of logical form!

“Peirce’s Puzzle”

- The following are equivalent in first order predicate logic:
 - (A) $(\exists x\phi(x) \leftarrow \forall x\psi(x))$;
 - (B) $\exists x(\phi(x) \leftarrow \psi(x))$.

The Sweepstakes (Stephen Read)

- There are a thousand people who may participate;
- each participant brings in one dollar.

“The Puzzle”

- (1) Someone wins \$1.000 if everyone takes part.
- (2) Someone wins \$1.000 if he takes part.

(A) $(\exists x Wx \leftarrow \forall x Tx)$

(B) $\exists x(Wx \leftarrow Tx)$

- Arguably (1) \rightsquigarrow (A) and (2) \rightsquigarrow (B);
- but (A) \Leftrightarrow (B) and (1) $\not\Leftarrow$ (2).
- (2), and not (1), will prompt the question “Who?”, “Daisy?”

Support Solution

(1) Someone wins \$1.000 if everyone takes part.
 $(\exists x Wx \leftarrow \forall x Tx)$

- $\sigma \models_{\mathcal{M},g}$ (1) with the description given; but

(2) Someone wins \$1.000 if he takes part.
 $\exists x(Wx \leftarrow Tx)$

- $\sigma \models_{\mathcal{M},g,d}$ (2) iff $\sigma \models_{\mathcal{M},g[x/d]}$, $Wx \leftarrow Tx$

- σ supports (2) iff $(\forall w \in \sigma) d(w)$ wins in w if she takes part in w .

How can that be?

- ① Let's call d 'Daisy', to give her a name.
 - ② σ thinks that Daisy wins (*quantity).
 - ③ σ thinks that Daisy does not participate (*quantity).
 - ④ σ thinks that Daisy is a predestined winner[✓].
- This predicts everyone takes part, if Daisy does;
 - the sweepstakes is a frame up, or Daisy is a lucky duck;
 - or Daisy is a freaky concept.