Meaning, Reference and Modality

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Two Roles of Facts and Two Types of Information

- Stalnaker's two roles of facts in interpretation.
- Interpretation takes place in the world it is about.
- There is (update of) information about the world (US), and there is (update of) information about the discourse (DPL).

- Brown utters the words "Jones is raking the leaves." Brown uses the name "Jones" to refer to Smith.
- He told us that Jones is raking the leaves.

- Information about the information of others:
  - left for another occasion.

Two Theories of Dynamic Semantics

- In update semantics, interpretation is an update function over information states:
  - $s[\![\phi]\!]$, with information state s a set of valuations.
- In dynamic predicate logic, interpretation is a relation between variable assignments:
  - $g[\![\psi]\!]_M h$, with variable assignments g and h.
- Can be lifted to the format of an update function over information states:
  - $G[\![\psi]\!] = \{h \mid \exists g \in G \;\&\; g[\![\psi]\!]_M h\}$, with G a set of variable assignments.

Unfibrability

- The US inference relation is not reflexive and not monotone.
- The DPL inference relation is not transitive either.
- Three basic properties for Gabbay's (1999) combining ('fibring') logics.
- The systems are non-structural for different reasons.
  - US has global operators. ($\Diamond$ is not 'distributive'.)
  - DPL has non-persistent operators. ($\exists$ is 'destructive'.)
- The DPL format resists global interpretation.
- The US format resists distributive interpretation.

Possible Solutions

- Two-dimensional product of the two systems (van Eijck and Cepparello, 1994)
  - Resists the expression of uncertainty about the values of variables. $\exists x(x^2 = 4) \wedge \Diamond(x = -2) \models x = -2$
- Making US distributive after all. Non-well founded information states. (Unpub.)
- Make DPL persistent. (GSV, Vermeulen, Aloni, myself.)

Referent Systems and Possibilities

- A referent system r is an injection with
  - domain $X \subseteq V$ (variables) and
  - range $n = \{m \mid m < n\}$.
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	x	$-$	y	$\ldots$	z
n:	0	1	2	$\ldots$	$n-1$

- An assignment function g for r has
  - domain $range(r)$ and
  - domain D.
-

	x	$-$	y	$\ldots$	z
n:	0	1	2	$\ldots$	$n-1$
	a	a'	a''	$\ldots$	a'''

Extending a Referent System

- Extension $r + y$ of a referent system r with y has:
  - $domain(r + y) = domain(r) \cup \{y\}$;
  - $range(r + y) = range(r) + 1$;
  - $(r + y) = r[y/range(r)]$.

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	x	$-$	y	$\dots$	z
n:	0	1	2	$\dots$	$n-1$

$\Rightarrow$

	x	$-$	$-$	$\dots$	z	y
$n+1$:	0	1	2	$\dots$	$n-1$	n

- r extends into r', $r \leq r'$, iff
  - $r + x \dots = r'$, for some sequence of variables $x \dots$.

Possibilities and Information States

- A possibility is a triple $i = \langle r, g, w \rangle$ with
  - w an interpretation function.
- An information state is a set of possibilities with the same referent system.
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	x	$-$	y	$\dots$	z
n:	0	1	2	$\dots$	$n-1$
w_1	a	a'	a''	$\dots$	a'''
w_2	b	b'	b''	$\dots$	b'''
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$

Extension of Information

- Updating consists in extending an information state, with world information and discourse information:
  - elimination of possibilities and
  - addition of new discourse items.

- i extends into a possibility i', $i \leq i'$, iff
  - $r \leq r', g \subseteq g', w = w'$.
  - (We may lose information captured by r, but not that captured by g.)

- $i[x/d] = \langle r + x, g[range(r)/d], w \rangle$,
  $s[x/d] = \{i[x/d] \mid i \in s\}$.

- s' extends an information state s iff:
  - $\forall i' \in s' \exists i \in s$: $i \leq i'$

DMPL Interpretation of Terms (Including Demonstratives)

- $i(c) = w(c)$ for constants c;
  $i(x) = g(r(x))$ for variables x;
  $i(d) = d$ for demonstratives d.
- "That's right, John himself, right there, trapped in a proposition." (David Kaplan)
- "That's right, John himself, right there, trapped in a formula."(Paul Dekker).

DMPL Interpretation of Formulas

- $s[\![Rt_1, \dots, t_n]\!] = \{i \in s \mid \langle i(t_1), \dots, i(t_n\rangle \in i(R)\}$
  $s[\![\neg\phi]\!] = \{i \in s \mid \neg\exists i'\text{: } i \leq i' \ \&\ i' \in s[\![\phi]\!]\}$
  $s[\![\phi \wedge \psi]\!] = s[\![\phi]\!][\![\psi]\!]$
  $s[\![\Diamond\phi]\!] = \{i \in s \mid s[\![\phi]\!] \neq \emptyset\}$
  $s[\![\exists x\phi]\!] = \bigcup_{d \in D}(s[x/d][\![\phi]\!])$

≫ not: $s[\![\exists x\phi]\!] = (\bigcup_{d \in D} s[x/d])[\![\phi]\!]$

Support and Consistency

- ϕ is consistent iff $s[\![\phi]\!] \neq \emptyset$ for some s;
- s subsists in s', $s \sqsubseteq s'$, iff $\forall i \in s \exists i' \in s'$: $i \leq i'$
  - (only discourse information added);
- s supports ϕ, $s \models \phi$, iff $s \sqsubseteq s[\![\phi]\!]$;
- ϕ is coherent iff $s \models \phi$ for some $s \neq \emptyset$
- ϕ is valid iff $s \models \phi$ for all s.

Typical US and DPL Facts

- $(\phi \wedge (\psi \wedge \chi)) \Leftrightarrow ((\phi \wedge \psi) \wedge \chi)$
- $\phi \not\Leftrightarrow (\phi \wedge \phi)$
- $(\phi \wedge \psi) \not\Leftrightarrow (\psi \wedge \phi)$
- $(\Diamond p \wedge \neg p)$ is consistent;
  $(\neg p \wedge \Diamond p)$ is not
  neither is$((\neg p \wedge \Diamond p) \wedge (\neg p \wedge \Diamond p))$
- $(\exists x(Mx \wedge Rx) \wedge Wx) \;\Leftrightarrow$
  $\exists x((Mx \wedge Rx) \wedge Wx) \;\not\Leftrightarrow$
  $(Wx \wedge \exists x(Mx \wedge Rx))$

Identifications

- Frank Veltman's JoLLI dots. Suppose:
  - $\exists x(a = x) \wedge \exists y(b = y) \wedge (x \neq y) \wedge \forall z((x = z) \vee (y = z))$;
  - $s[\![\Diamond(d_1 = a); \Diamond(d_1 = b)]\!] \not\models \forall z \Diamond(d_1 = z)$
  - $s[\![\Diamond(d_1 = a); \Diamond(d_2 = a)]\!] \models \forall z \Diamond(z = a)$.
- The following is consistent no matter what type of term t is
  - $\Diamond Pt \wedge \Diamond \neg Pt$;
- but with an identity statement (cf., belief puzzles, Hesperus, Ortcutt)
  - $\Diamond(t_1 = t_2) \wedge \Diamond \neg(t_1 = t_2)$
- is consistent if t_1 or t_2 is a name or free variable,
- is inconsistent if t_1 and t_2 are demonstratives or bound variables.

Universal Instantiation?

- Universal instantiation is restricted:
  - $\forall x \phi \models [t/x]\phi$ if t is a demonstrative or a 'bound variable',
  - not if t is a free variable or name.

(1) Anybody on this party here might *not* be Dr. Livingstone.
But not Dr. Livingstone himself, of course.
$\forall x \Diamond (x \neq L) \wedge \neg \Diamond (L \neq L)$ is consistent

Existential Generalization?

- Existential generalization is restricted:
  - $[t/x]\phi \models \exists x\phi$ if t is a demonstrative or a 'bound variable',
  - not if t is a free variable or name.

(2) Of course I know Dr. Livingstone is Dr. Livingstone.
But he can be anybody here.
$\Box(L = L) \wedge \neg\exists x\Box(x = L)$ is consistent

Egli's Theorem revisited

- $\exists x Px \wedge \Diamond Qx \not\Leftrightarrow \exists x(Px \wedge \Diamond Qx)$
- $\exists x Px \rightarrow \Diamond Qx \not\Leftrightarrow \forall x(Px \rightarrow \Diamond Qx)$
- $\exists x\phi \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$, provided

there are no free occurrences of x in ϕ $(< DPL)$;

$\top$ (DPL);

there are no free occurrences of x under modals in ϕ $(\geq DMPL)$.

The Broken Vase, Scene 1

"You and your spouse have three sons. One of them broke a vase. Your spouse is very anxious to find out who did it. Both you and your spouse know that your eldest didn't do it, he was playing outside when it must have happened. Actually, you are not interested in the question who broke the vase. But you are looking for your eldest son to help you do the dishes. He might be hiding somewhere.

In search for the culprit, your spouse has gone upstairs. Suppose your spouse hears a noise coming from the closet. If it is the shuffling of feet, your spoose will know that someone is hiding in there, but will not be able to exclude any of your three sons."

The Situation

- So you are Donald Duck, the information state of your spouse is called Daisy, your three sons are Huey, Dewie and Louie, the last one of which was decently playing outside when *it* happened, so:
- daisy $= \{w_1, \ldots, w_6\}$ where

	Guilty	Closet
w_1:	h	h
w_2:	h	d
w_3:	h	l
w_4:	d	h
w_5:	d	d
w_6:	d	l

The Support

- daisy $\models$ Huey may be guilty, Dewie may be guilty
- daisy $\not\models$ Louie may be guilty
- daisy $\models$ Huey, Dewie or Louie is in the closet, so
  - daisy $\models$ Somebody is in the closet, and he may be Huey, so
  - daisy $\models$ Somebody is in the closet, and he may be guilty, but
  - daisy $\not\models$ Somebody is in the closet who is Huey or Dewie, so
  - daisy $\not\models$ Somebody is in the closet who may be guilty

Formally

- $daisy[\![\exists x Cx]\!] =$ (table below) $\models \Diamond Gx$

	Guilty	Closet	x
w_1:	h	h	h
w_2:	h	d	d
w_3:	h	l	l
w_4:	d	h	h
w_5:	d	d	d
w_6:	d	l	l

- $daisy \not\models daisy[\![\exists x(Cx \wedge \Diamond Gx)]\!] =$

	Guilty	Closet	x
w_1:	h	h	h
w_2:	h	d	d
w_4:	d	h	h
w_5:	d	d	d

The Broken Vase, Scene 2

(3) If a boy is hiding in the closet he might be guilty.

- daisy $\models (\exists x Cx \rightarrow \Diamond Gx)$

(4) Any boy who is hiding in the closet might be guilty.

- daisy $\not\models \forall x(Cx \rightarrow \Diamond Gx)$

- $(\exists x Cx \rightarrow \Diamond Gx)$: if there is a C then maybe a C did it
- $\forall x(Cx \rightarrow \Diamond Gx)$: every C is suspect

Conceptual Covers by Maria Aloni

- Pointing in a situation,
- and referring to individuals by name.

- These are just two of many alternative ways of accessing individuals in a given domain.

- Every individual may in each real situation appear in a different guise.

- Questions of identification, *Wh*-questions, and support issues.

The ‘Aloni’ Sequence

(5) Someone might be the culprit. She is not the culprit.
$(\exists x \Diamond Px \wedge \neg Px)$ $(= \phi \wedge \psi)$

- This is supportable in DMPL.

- let:

	P
v	$\{a\}$
w	$\{b\}$

  and let: $s = \{v, w\}$

- $s[\![\exists x \Diamond Px]\!] = \bigcup_{d \in D} s[x/d][\![\Diamond Px]\!] =$
  $s[x/a][\![\Diamond Px]\!] \cup s[x/b][\![\Diamond Px]\!] = s[x/a] \cup s[x/b]$
- $(s[x/a] \cup s[x/b])[\![\neg Px]\!] = (\{v\}[x/b] \cup \{w\}[x/a])$
  $s \sqsubseteq (\{v\}[x/b] \cup \{w\}[x/a])$.

- Some state (s) supports the formula so it is coherent in DMPL.

Interpretation under Cover

- Maria Aloni saw the problem as a student; studied and solved it as a PhD student; extended the solution to two other problems; and won the FoLLI dissertation price.

- Solution (more later): Bound and free variables range, equally, not over individuals, but over individual concepts from a conceptual cover.
- The Aloni sequence is satisfiable, but not supportable in CC
- Intuitively, the speaker 'talks' about the person, whoever it is, who didn't do it ($\neg Px$); but that's not the person who might have done it ($\Diamond Px$)
- The concept of x which would support $\Diamond Px$ cannot be the one under which the speaker also supports $\neg Px$.

Egli's Theorem Revisited

- Egli's theorem is rendered valid again.
- *daisy* $\models_{c_1} \exists x \Box \neg Gx$ under the cover c_1 consisting of
  - 'Huey',
  - 'Dewie' and
  - 'Louie'
- *daisy* $\models_{c_2} \forall x \Diamond Gx$ under the cover c_2 consisting of
  - 'the person heard in the closet'
  - e.g., 'the largest one not in the closet', and
  - 'the smallest one not in the closet'

(6) There is someone in the closet and he might have done it.

- *daisy* $\not\models_{c_1} (\exists x Cx \wedge \Diamond Gx)$
- *daisy* $\models_{c_2} (\exists x Cx \wedge \Diamond Gx)$