Meaning, Reference and Modality

Paul Dekker

ILLC/Department of Philosophy
Universiteit van Amsterdam
p.j.e.dekker@uva.nl

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Some Motivating Examples

1. A dog entered the garden. It is barking.
   ?It is barking. A dog entered the garden.
2. If a cat is hungry it usually meows.
   ?It usually meows if a cat is hungry.
3. Rebecca married Thomas. She regrets that she married him.
   Rebecca regrets that she married Thomas. ?She married him.
4. Bob left. Conny started to cry. (Weak-hearted Conny!)
   Conny started to cry. Bob left. (Hard-hearted Bob!)
5. Max turned off the light. The room was pitch dark.
   ?The room was pitch dark. Max turned off the light.
6. I tell you your wife is cheating on you; now you know it.
   Now you know your wife is cheating on you; I tell you.
7. If Isabel is in the bathroom, Petra might be there, too.
   If Isabel is in the bathroom and nobody else is, Petra might be there, too.

Dynamic Predicate Logic

"restricts the dynamics of interpretation to that aspect of the meaning of sentences that concerns their potential to 'pass on' possible antecedents for subsequent anaphors, within and across sentence boundaries." (Groenendijk and Stokhof, p. 43-44)

Dynamic Binding

- A man is riding through the park. He is whistling.
- A man who is riding through the park is whistling.

≫ $(\exists x(Mx \wedge Rx) \wedge Wx) \Leftrightarrow \exists x((Mx \wedge Rx) \wedge Wx)$

"Egli's Theorem"

- $(\exists x\phi \wedge \psi) \equiv \exists x(\phi \wedge \psi)$ (unconditionally!)

"Egli's Corollary"

- $(\exists x\phi \rightarrow \psi) =_{df} \neg(\exists x\phi \wedge \neg\psi) \equiv_{egli}$
  $\neg\exists x(\phi \wedge \neg\psi)) =_{df} \forall x\neg(\phi \wedge \neg\psi) =_{df} \forall x(\phi \rightarrow \psi)$

- If a farmer owns a donkey, he beats it.
- Every farmer beats every donkey he owns.

≫ $(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow Bxy) \Leftrightarrow$
$\forall x(Fx \rightarrow \forall y((Dy \wedge Oxy) \rightarrow Bxy))$

Dynamic Predicate Logic

- indefinite noun phrases (existentially quantified expressions) set up discourse referents
  - introduce file cards, declare variables, ...
- pronouns (free variables) refer back to them

- both are associated with variables
- states contain information about the values of variables
- these information states get updated in discourse
- no substantial information about the world

- interpretation in ordinary PL: sets of assignments
  in DPL: sets of pairs of assignments; input-output pairs

Once Upon a Time, in the Middle of a Discourse . . .

- $\checkmark_0$ Mary borrowed a copy of *Naming and Necessity*$_x$ from a professor in linguistics$_y$.$\checkmark_1$ The pages were covered with comments and exclamations.$\checkmark_2$ He$_y$ must have been studying it$_x$ intensively.$\checkmark_3$
- at checkpoint $\checkmark_1$ x is a copy of *Naming and Necessity* Mary borrowed from a professor in linguistics
- at checkpoint $\checkmark_3$ x is a copy of *Naming and Necessity* Mary borrowed from a professor in linguistics (. . .) who must have studied it intensively

A Procedural Look upon FOPL

- $\models_{g^{\checkmark 0}} \exists x^{\checkmark 1}(CNx^{\checkmark 2} \wedge \exists y^{\checkmark 3}(PLOyx^{\checkmark 4} \wedge BORmxy^{\checkmark 5}))^{\checkmark 6}$
- $CNx :=$ x is a copy of Naming and Necessity
  $PLOyx :=$ y is a professor in Linguistics who owns x
  $BORmxy :=$ m borrowed x from y

- $\checkmark_1$: assign random value to x
  $\checkmark_2$: test $g'(x) \in E(CN)$
  $\checkmark_3$: assign random value to y
  $\checkmark_4$: test $\langle g''(y), g'(x)\rangle \in E(PLO)$
  $\checkmark_5$: test $\langle m, g'(x), g''(y)\rangle \in E(BOR)$
  $\checkmark_6$: 1 if you get here, 0 if you can't

- $\checkmark_6$ *DPL*: be happy if you get here ; –)
  and do not forget about x and y: remember g''

Notation

- h is an assignment verifying ϕ under assignment g
  - $h \models_g \phi$ (GAMUT)
- the input-output pair $\langle g, h \rangle$ is in the interpretation of ϕ
  - $\langle g, h \rangle \in [\![\phi]\!]$ (DPL)
- relative to input assignment g, the interpretation of ϕ may deliver output assignment h
  - $g[\![\phi]\!]h$ (here)

DPL Semantics

Interpretation

- $g[\![Rx_1 \ldots x_n]\!]h$ iff $g = h$ & $\langle g(x_1), \ldots, g(x_n)\rangle \in E(R)$
  $g[\![\neg\phi]\!]h$ iff $g = h$ & $\neg\exists h$: $g[\![\phi]\!]h$
  $g[\![\exists x\phi]\!]h$ iff $\exists k$: $g[x]k[\![\phi]\!]h$
  $g[\![\phi \wedge \psi]\!]h$ iff $\exists k$: $g[\![\phi]\!]k[\![\psi]\!]h$

Entailment

- $\phi_1, \ldots, \phi_n \models \psi$ iff
  $\forall g, k$: if $g[\![\phi_1 \wedge \ldots \wedge \phi_n]\!]k$, then $\exists h$: $k[\![\psi]\!]h$
  (for all models M of course)

A Farmer doesn't Own a Donkey. He Owns a Horse.

$$
\begin{array}{llllll}
g[\![\exists x\,(Fx \wedge \neg\exists y(Dy \wedge Oxy)) & \wedge & \exists z(Hz \wedge Oxz)]\!]h & \\
g[\![\exists x\,(Fx \wedge \neg\exists y(Dy \wedge Oxy))]\!] & k & [\![\exists z(Hz \wedge Oxz)]\!]h & \text{(fr sm } k\text{)} \\
g[x]l\,[\![Fx \wedge \neg\exists y(Dy \wedge Oxy)]\!] & k & [z]\ m\,[\![Hz \wedge Oxz]\!]h & \text{(fr sm } l,k,m\text{)} \\
g[x]l\,[\![Fx]\!]\ l'\ [\![\neg\exists y(Dy \wedge Oxy)]\!] & k & [z]\ m\,[\![Hz]\!]\ m'\ [\![Oxz]\!]h & \text{(fr sm } l,l',k,m,m'\text{)} \\
g[x]k[\![Fx]\!]\ k\ [\![\neg\exists y(Dy \wedge Oxy)]\!] & k & [z]\ h\,[\![Hz]\!]\ h\ [\![Oxz]\!]h & \text{(fr sm } k\text{ :)} \\
\quad k\ [\![\exists y(Dy \wedge Oxy)]\!] & m & & \text{(fr no } m\text{)} \\
\quad k\ [y]\ m\ [\![Dy]\!]\ m\ [\![Oxy]\!]\ m & & & \text{(fr no } m\text{)}
\end{array}
$$

- $\exists a$: $a \in I(F)$ and $\neg\exists b(b \in I(D)\ \&\ \langle a,b\rangle \in I(O))$ and $\exists c$: $c \in I(H)$ and $\langle a,c\rangle \in I(O)$ and $h = g[x/a][z/c]$

Dynamic Entailment

- An example from Peter Geach, from Peter Strawson
- A man has just drunk a pint of sulphuric acid.
  Nobody who drinks sulphuric acid lives through the day.
  Very well then, *he* won't live through the day.
  $\exists x(Mx \wedge DPoSAx), \neg\exists z(DPSDz \wedge LTDz) \models \neg LTDx.$

Normal Binding Form ($\bullet^\clubsuit$)

- define the normal binding form $\phi^\clubsuit$ of ϕ

Requirement

- $\phi^\clubsuit \Leftrightarrow_{DPL} \phi$ and
- $\models_{DPL,g} \phi^\clubsuit$ iff $\models_{FOPL,g} \phi^\clubsuit$

Definition

$(Rt_1 \ldots t_n)^\clubsuit$	$= Rt_1 \ldots t_n$	$(Rt_1 \ldots t_n \wedge \psi)^\clubsuit$	$= (Rt_1 \ldots t_n)^\clubsuit \wedge (\psi)^\clubsuit$
$(\neg\phi)^\clubsuit$	$= \neg(\phi)^\clubsuit$	$(\neg\phi \wedge \psi)^\clubsuit$	$= (\neg\phi)^\clubsuit \wedge (\psi)^\clubsuit$
$(\exists x\phi)^\clubsuit$	$= \exists x(\phi)^\clubsuit$	$(\exists x\phi \wedge \psi)^\clubsuit$	$=^* (\exists x(\phi \wedge \psi))^\clubsuit$
		$((\phi \wedge \psi) \wedge \chi)^\clubsuit$	$= (\phi \wedge (\psi \wedge \chi))^\clubsuit$

≫ * (Egli's theorem) is the only real difference

≫ ≫ mission completed!!!

≫ ≫≫ or not???

Cliff Hanger

- DPL is PL plus Egli's Theorem
  - *without* "if x does not occur free in ψ"

Characteristic Properties

- $\neg\phi$ annihilates any binding potential of ϕ
  - $^{?}(\neg\exists x(Mx \wedge Rx) \wedge Wx)$
  - No man is riding through the park. $^{?}$He is whistling.
- $\rightarrow$, $\forall x$, and $\vee$ definable in terms of $\neg$, $\wedge$ and $\exists x$
- $\forall x\phi$, $(\chi \rightarrow \phi)$ and $(\chi \vee \phi)$ annihilate the binding potential of ϕ
  - Every student has a cat. *It drives me crazy.
  - If Ben graduated, he wrote a thesis. $^{?}$It is very lengthy.
  - Carl went to the party, or somebody forbid him. $^{?}$It was his Mom.

Characteristic Properties

- $\phi \neq \neg\neg\phi$
  - There is a Welsh doctor in London. She can help you.
  - It is not so that there is no Welsh doctor in London. $^?$She can help you.
- $\wedge$ and $\exists x$ *not* definable in terms of $\neg$, $\rightarrow$, $\forall x$, and $\vee$
  - A man is riding through the park. he is whistling.
  - Not every man is not riding in the park. $^?$He is whistling.

Characteristic Properties

- conjunction is not (always) commutative or idempotent
  - $(Fx \wedge \exists xGx) \not\Leftrightarrow (\exists xGx \wedge Fx)$
- difference between syntactic scope and semantic binding
  - $(\exists x(Mx \wedge Wx) \wedge WHx)$; the fourth occurrence of x is not in the scope of the quantifier, but it is bound by it
  - $(\exists x((Mx \wedge \exists xWx) \wedge WHx)$; the last occurrence of x is in the scope of the first quantifier, but bound by the second
- α-conversion is blocked
  - $(\exists x(Mx \wedge Rx) \wedge Wx) \not\Leftrightarrow (\exists y(My \wedge Ry) \wedge Wx)$

Non-Monotone Entailment

The inference relation is dynamic, hence sensitive to order (and number) of premises

(8) $\exists x Fx \models Fx$

(9) $\exists x Fx, \exists x \neg Fx \not\models Fx$

(10) $\exists x \neg Fx, \exists x Fx \models Fx$

(11) Let x be even. So, x is even.

(12) Let x be even. Now let x be odd. $^{?}$So, x is even.

(13) Let x be odd. Now let x be even. So, x is even.

Non-Reflexive Entailment

One and the same formula may come to mean something different on different locations in a discourse.

- $(Cx \wedge \exists x Tx) \not\models (Cx \wedge \exists x Tx)$

(14) It is a chair and there is a table. It is a chair and there is a table.

(15) Ervik is first now, but Veldkamp takes the lead. ?So Ervik is first now, but Veldkamp takes the lead.

Non-Transitive Entailment

(16) Aristotle: All A are B, all B are C; so, all A are C.

(17) van Benthem:
All men who have a house are men who have a garden.
All men who have a garden sprinkle it on Saturdays.
$^{?}$So, all men who have a house sprinkle it on Saturdays.

(18) It is not the case that there is no Welsh doctor in London.
Hence, there *is* a doctor in London who is Welsh.
So *he* is Welsh.
$\neg\neg\exists x Px \models \exists x Px \models Px.$

(19) It is not the case that there is no Welsh doctor in London.
$^{?}$So *he* is Welsh.
$\neg\neg\exists x Px \not\models Px.$

In Conclusion

- DPL extends FOPL with Egli's theorem, and nothing more
- it has the expressive power of FOPL
- it has computational / compositional benefits
- countenanced by serious logical complications
- which are naturally motivated though
- from a dynamic semantic perspective

Complications with Plurals

- Bob and Carol went to play bridge with Ted and Alice. They had a wonderful evening.
- Seven pupils and four teachers wrote five ballads and some rhymes. They performed them at an evening during the spring holiday.
- Every boy who brought a Christmas gift for a girl in his class asked her deskmate to wrap it.

Solutions with Plurals

- Adrian Brasoveanu, "Decomposing Modal Quantification", 2009

- Every^{u_1} person who buys a^{u_2} book on amazon.com and has a^{u_3} credit card uses it$_{u_3}$ to pay for it$_{u_2}$.
- $max^{u_1}(_{u_1}([person\{u_1\}] \wedge$
  $max^{u_2}([book\{u_2\}, buy\{u_1, u_2\}]) \wedge$
  $max^{u_3 \sqsubseteq r}([card\{u_3\}, have\{u_1, u_3\}]))) \wedge$
  $DIST_{u_1}([sing\{u_3\}, sing\{u_2\}, use - to - pay\{u_1, u_3, u_2\}])$

- I am not going to explain this
- it requires a lot of structure
- but only DPL-conjunction

Complications with Modals

A man cannot live without joy. Therefore, when he is deprived of true spiritual joys, it is necessary that he become addicted to carnal pleasures.
(Thomas Aquinas, rendered by Adrian Brasoveanu, 2009.)

- If^{p_2} a^{u_1} man is alive, he$_{u_1}$ must$^{p_3 \sqsubseteq p_2}_{p_1,\beta,\omega}$ have a^{u_2} pleasure. Therefore$^{p_4 \sqsubseteq p_1}_{p^*,\beta^*,\omega^*}$, if$^{p_5 \sqsubseteq p_2}$ he$_{u_1}$ doesnt have any^{u_3} spiritual pleasure, he$_{u_1}$ must$^{p_6 \sqsubseteq p_5}_{p_1,\beta,\omega}$ have a^{u_4} carnal pleasure.
- logical translation omitted

Solutions with Modals

- Maria Bittner, "Counterfactuals as real attitudes", 2009

• indicative conditional with branching (cf. Stalnaker1975, Thomason 1984)
• future interpretation of nonpast event verbs (cf. Condoravdi 2002)
• temporal recentering by event verbs (cf. Kamp 1979, Webber 1988)
• anaphoric parallel between tenses and pronouns (cf. Partee 1973)

E4. *If we adopt a baby, we will give it a good home.*
IF 1PL adopt.NPST A baby 1PL FUT.VIV give 3SG A good home
IF [1PL adopt-
P[w| w ∈ **d**Ω]; [**a**| (AGT **d**ε ⊂ **a**)°]; [e| *adopt*⟨dω, e: AGT, EXP⟩];
-NPST
P[**t**| (ϑ(**d**ω, **d**ε) ≤ **t**)°]; [⊃⟨dω: dε, **d**τ⟩]; [(CTR dε = **d**α)°];
[**t**| (**t** ⊂ ϑ(dω, CON dε))°];
A baby
[a| (SG a), (EXP dε = a)°]; [(*baby*⟨dω: ϑ(ω: dε), dα⟩];
] 1PL
[**p**]; [**d**Ω = dω{|$_{\mathbf{d}\alpha}$}]; [(AGT **d**ε ⊂ **d**α)°];
FUT.
P[(ϑ(**d**ω, **d**ε) < **d**τ)°]; [e]; [⊃⟨dω: dε, **d**τ⟩]; [(CTR dε = **d**α)°];
[**t**| (**t** ⊂ ϑ(dω, CON dε))°];
.VIV
P[Q| (**d**Ω ⊆ $\mathbf{d}\Omega_2$)°, ($\mathbf{d}\Omega_2$ ∈ Q)°, (Q ⊆ BEL+DES(**d**ω, **d**ε))°];
[give
[*give*⟨dω, dε: AGT, EXP, CTR′⟩];
3SG
P[(dα ∅ (AGT **d**ε + EXP **d**ε))°, (SG dα)°]; [EXP dε = dα];
A good
[b| (SG b)°, (CTR′ dε = b)°]; [*good.for*⟨dω: ϑ(ω: CON dε), dβ, dα⟩];
home
[*home.of*⟨dω: ϑ(ω: CON dε), dβ, dα⟩];
] **.** (S-final prosody)
[MIN(dΩt, **d**Ω) ⊆ dω{|$_{\mathbf{d}\alpha, \mathbf{d}\omega, d\Omega t}$}]; [**p**]; [**d**Ω = **d**ω{|}]

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