Meaning, Reference and Modality

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Stalnaker's Assertion

- *Assertion* and *On the Representation of Context*
  - Do not use the version in the Peter Cole volume, but the one in Robert Stalnaker, 1999, *Context and Content*, Oxford University Press, pp. 78–95, werkkast 457.107.
- 'Lewis' philosophy': systematize our pre-philosophical opinions.
- Stating and fleshing out four 'truisms'.
- How far can you can follow and keep on agreeing?

Stalnaker's First Three Truisms

1. Assertions have content.
   - Propositions, "a representation of the world as being a certain way", sets of indices.
2. Assertions are made in a context.
   - Basically the same indices.
3. Sometimes the content of an assertion is dependent on the context in which it is made.
   - Propositional concepts, functions from indices to propositions.

(1) Someday you will be grateful for what I am doing now.

(2) A child was born which would/will rule the world.

(3) I am the speaker now.

(4) Necessarily I am the speaker.

(5) If Obama had been baptized with the name 'Osama', he wouldn't have become the president.

Some Terminology

- Propositional concepts, propositions, extensions (in reverse order).
- The extension (truth value) of an expression is determined by the evaluation in a context (index of evaluation) of its content (proposition expressed).
- The content of an expression (proposition) is determined by its meaning (propositional concept) in a context (index of use).
- One and the same world may play two different roles.

Somewhat More Formal

- Suppose
  - $\models_i$ David utters "Bob is mad." and
    $\models_i$ "Bob" refers to, say, b;
- then
  - $\models_i$ David expresses $p = \{j \mid I_i(Bob) = b \in I_j(M)\}$ and
    p is true at i iff $i \in p$ iff $I_i(Bob) = b \in I_i(M)$.

- Suppose
  - $\models_i$ David utters "Possibly Bob is mad." and
    $\models_i$ "Bob" refers to, say, b;
- then
  - $\models_i$ David expresses $q = \{k \mid \exists j\colon kRj \wedge j \in p\}$ and
    q is true at i iff $\exists j\colon iRj$ & $I_i(Bob) = b \in I_j(M)$.

- Two indices relevant.

Two-Dimensional Matrices

"Bob is mad."

Two-Dimensional Matrices

"Bob is mad."	
i	
j	
k	

Two-Dimensional Matrices

"Bob is mad."	
i	
j	
k	
index of use ↑	

Two-Dimensional Matrices

"Bob is mad."	i	j	k	
i				
j				
k				
index of use ↑				

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i				
j				
k				
index of use ↑				

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i				"Bob" denotes b
j				"Bob" denotes b
k				"Bob" denotes d
index of use ↑				

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i				"Bob" denotes b
j				"Bob" denotes b
k				"Bob" denotes d
index of use ↑				relevant facts of use ↑

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i				"Bob" denotes b
j				"Bob" denotes b
k				"Bob" denotes d
index of use ↑	Sb, Md	Mb, Sd	Mb, Sd	relevant facts of use ↑

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i				"Bob" denotes b
j				"Bob" denotes b
k				"Bob" denotes d
index of use ↑				relevant facts of use ↑
	Sb, Md	Mb, Sd	Mb, Sd	← relevant evaluation facts

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i	F			"Bob" denotes b
j				"Bob" denotes b
k				"Bob" denotes d
index of use ↑				relevant facts of use ↑
	Sb, Md	Mb, Sd	Mb, Sd	← relevant evaluation facts

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i	F	T		"Bob" denotes b
j				"Bob" denotes b
k				"Bob" denotes d
index of use ↑				relevant facts of use ↑
	Sb, Md	Mb, Sd	Mb, Sd	← relevant evaluation facts

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i	F	T	T	"Bob" denotes b
j				"Bob" denotes b
k				"Bob" denotes d
index of use ↑				relevant facts of use ↑
	Sb, Md	Mb, Sd	Mb, Sd	← relevant evaluation facts

Two-Dimensional Matrices

"Bob is mad."	i	j	k	$\leftarrow$ index of evaluation
i	F	T	T	"Bob" denotes b
j	F	T	T	"Bob" denotes b
k				"Bob" denotes d
index of use $\uparrow$				relevant facts of use $\uparrow$
	Sb, Md	Mb, Sd	Mb, Sd	$\leftarrow$ relevant evaluation facts

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i	F	T	T	"Bob" denotes b
j	F	T	T	"Bob" denotes b
k	T			"Bob" denotes d
index of use ↑				relevant facts of use ↑
	Sb, Md	Mb, Sd	Mb, Sd	← relevant evaluation facts

Two-Dimensional Matrices

"Bob is mad."	i	j	k	$\leftarrow$ index of evaluation
i	F	T	T	"Bob" denotes b
j	F	T	T	"Bob" denotes b
k	T	F		"Bob" denotes d
index of use $\uparrow$				relevant facts of use $\uparrow$
	Sb, Md	Mb, Sd	Mb, Sd	$\leftarrow$ relevant evaluation facts

Two-Dimensional Matrices

"Bob is mad."	i	j	k	← index of evaluation
i	F	T	T	"Bob" denotes b
j	F	T	T	"Bob" denotes b
k	T	F	F	"Bob" denotes d
index of use ↑				relevant facts of use ↑
	Sb, Md	Mb, Sd	Mb, Sd	← relevant evaluation facts

Modalities in Use

'Bob is mad.'	i	j	k	
i	F	T	T	'B' $\rightsquigarrow b$
j	F	T	T	'B' $\rightsquigarrow b$
k	T	F	F	'B' $\rightsquigarrow d$
	Sb, Md	Mb, Sd	Mb, Sd	

(6) Possibly, Bob is mad.
True in i and j, because Mb in j and k; true in k, since Md in i.

'David is called 'Bob'.'	i	j	k	
i	F	F	T	'D' $\rightsquigarrow d$
j	F	F	T	'D' $\rightsquigarrow d$
k	F	F	T	'D' $\rightsquigarrow d$
	'B' $\rightsquigarrow b$	'B' $\rightsquigarrow b$	'B' $\rightsquigarrow d$	

(7) Possibly, David is called 'Bob'.
True in i, j and k, because 'Bob' denotes David in k.

Modalities in Use

'David is Bob.'	i	j	k	
i	F	F	F	'D' $\rightsquigarrow d$ & 'B' $\rightsquigarrow b$
j	F	F	F	'D' $\rightsquigarrow d$ & 'B' $\rightsquigarrow b$
k	T	T	T	'D' $\rightsquigarrow d$ & 'B' $\rightsquigarrow d$

(8) Necessarily, Bob is not David.
True in i and j, false in k.

'David is called 'Bob'.'	i	j	k	
i	F	F	T	'D' $\rightsquigarrow d$ & 'B' $\rightsquigarrow b$
j	F	F	T	'D' $\rightsquigarrow d$ & 'B' $\rightsquigarrow b$
k	F	F	T	'D' $\rightsquigarrow d$ & 'B' $\rightsquigarrow d$
	'B' $\rightsquigarrow b$	'B' $\rightsquigarrow b$	'B' $\rightsquigarrow d$	

(9) If David had been called 'Bob', Bob would not have been David.
True in i and j, false in k.

Constant Patterns

	i	j	k	
i	X	Y	Z	
j	X	Y	Z	
k	X	Y	Z	

- All rows the same: context-independence.

	i	j	k	
i	X	X	X	
j	Y	Y	Y	
k	Z	Z	Z	

- All columns the same: necessity.

New Operators

- $[\![\neg\phi]\!](i,j) = (1 - [\![\phi]\!](i,j))$
  - determined by points only
- $[\![\Diamond\phi]\!](i,j) = 1$ iff there is k: $[\![\phi]\!](i,k) = 1$
  - determined by whole rows
- $[\![\dagger\phi]\!](i,j) = [\![\phi]\!](j,j)$
  - determined by diagonals, projected on the rows
  - removes dependence on context of use

The Decontextualizing Dagger

'Bob is Mad.'	i	j	k	
i	F	T	T	'Bob' $\rightsquigarrow b$
j	F	T	T	'Bob' $\rightsquigarrow b$
k	T	F	F	'Bob' $\rightsquigarrow d$
	Sb, Md	Mb, Sd	Mb, Sd	

'† Bob is Mad.'	i	j	k	
i	F			'Bob' $\rightsquigarrow b$
j		T		'Bob' $\rightsquigarrow b$
k			F	'Bob' $\rightsquigarrow d$
	Sb, Md	Mb, Sd	Mb, Sd	

The Decontextualizing Dagger

'Bob is Mad.'	i	j	k	
i	F	T	T	'Bob' $\rightsquigarrow b$
j	F	T	T	'Bob' $\rightsquigarrow b$
k	T	F	F	'Bob' $\rightsquigarrow d$
	Sb, Md	Mb, Sd	Mb, Sd	

'† Bob is Mad.'	i	j	k	
i	F	T	F	'Bob' $\rightsquigarrow b$
j	F	T	F	'Bob' $\rightsquigarrow b$
k	F	T	F	'Bob' $\rightsquigarrow d$
	Sb, Md	Mb, Sd	Mb, Sd	

The Actuality DDagger

- $[\![\ddagger\phi]\!](i,j) = [\![\phi]\!](i,i)$
  - diagonal projected on columns

-

'‡ Bob is the actual madman'	i	j	k	
i	F			'Bob' $\rightsquigarrow b$
j		T		'Bob' $\rightsquigarrow b$
k			F	'Bob' $\rightsquigarrow d$
	Sb, Md	Mb, Sd	Mb, Sd	

The Actuality DDagger

- $[\![\ddagger\phi]\!](i,j) = [\![\phi]\!](i,i)$
  - diagonal projected on columns

-

'‡ Bob is the actual madman'	i	j	k	
i	F	F	F	'Bob' $\rightsquigarrow b$
j	T	T	T	'Bob' $\rightsquigarrow b$
k	F	F	F	'Bob' $\rightsquigarrow d$
	Sb, Md	Mb, Sd	Mb, Sd	

The Fourth Truism, Dynamic Semantics?

- Acts of assertion affect, and are intended to affect, the context.
- "the context on which an assertion has its ESSENTIAL effect is not defined by what is presupposed before the speaker begins to speak, but will include any information which the speaker assumes his audience can infer from the performance of the speech act."
- "The fact that a speaker is speaking, saying the words he is saying in the way he is saying them, is a fact that is usually accessible to everyone present."
- "To make an assertion is to reduce the context set in a particular way, provided that there are no objections from the other participants in the conversation."

Speaker Presupposition and Context Sets

- "Presuppositions are what is taken by the speaker to be the COMMON GROUND of the participants in the conversation, what is treated as their COMMON KNOWLEDGE or MUTUAL KNOWLEDGE."
- "(. . .) the more fundamental way of representing the speaker's presuppositions is (. . .) as a set of possible worlds, the possible worlds compatible with what is presupposed."
- "The presuppositions define the limits of the set of alternative possibilities among which speakers intend their expressions of propositions to distinguish."

- What the *speaker* assumes to be the common ground.
- (S)he and the other participants need not actually believe it.
- Contexts can be defective but they tend to adjust to an equilibrium.

Rational Principles of Communication

- " [T]he essential effect of an assertion is to change the presuppositions of the participants in the conversation by adding the content of what is asserted to what is presupposed. This effect is avoided only if the assertion is rejected."
- "essential conditions of rational communication"
  - contingency
  - definedness
  - constancy

A Decision Problem

-

"You are mad."	i	j	k	
i	F	T	T	'You' $\leadsto b$
j	F	T	T	'You' $\leadsto b$
k	T	F	F	'You' $\leadsto d$
	Sb, Md	Mb, Sd	Mb, Sd	

- "The problem is that since it is unknown which proposition it is that is expressed, the expression of it cannot do the job that it is supposed to do.'
- If the index of use is i or j, we should throw out i from the context set $\{i, j, k\}$;
- However, if the index of use is k, we should throw out j and k from the context set $\{i, j, k\}$.

Responses to Apparent Violations

- Revise the picture of the context set.
- Reinterpret or disambiguate the speaker.
- Accept the violation, and DAGGER the proposition.

(10) That is either Jeroen or Martin smoking.

- Violates the first and possibly the third principle.

"That is either Jeroen or Martin smoking."

-

'$((t = j \vee t = m) \wedge St)$'	i	j	k	
i	F	F	F	't' $\leadsto x \notin \{j, m\}$
j	F	T	F	't' $\leadsto j$
k	F	F	T	't' $\leadsto m$
	Sx	Sj	Sm	

-

'$\dagger((t = j \vee t = m) \wedge St)$'	i	j	k	
i	F			't' $\leadsto x \notin \{j, m\}$
j		T		't' $\leadsto j$
k			T	't' $\leadsto m$
	Sx	Sj	Sm	

"That is either Jeroen or Martin smoking."

-

'$((t = j \vee t = m) \wedge St)$'	i	j	k	
i	F	F	F	't' $\rightsquigarrow x \notin \{j, m\}$
j	F	T	F	't' $\rightsquigarrow j$
k	F	F	T	't' $\rightsquigarrow m$
	Sx	Sj	Sm	

-

'$\dagger((t = j \vee t = m) \wedge St)$'	i	j	k	
i	F	T	T	't' $\rightsquigarrow x \notin \{j, m\}$
j	F	T	T	't' $\rightsquigarrow j$
k	F	T	T	't' $\rightsquigarrow m$
	Sx	Sj	Sm	

Identity

(11) Hesperus is identical with Phosphorus.

- Violates the 1-st and 3-rd principle.

- Let v be Venus, and m is Mars.

-

'$h = p$'	i	j	
i	T	T	$h \rightsquigarrow v$ and $p \rightsquigarrow v$
j	F	F	$h \rightsquigarrow v$ and $p \rightsquigarrow m$
	$v \neq m$	$v \neq m$	

-

'$\dagger(h = p)$'	i	j	
i	T		$h \rightsquigarrow v$ and $p \rightsquigarrow v$
j		F	$h \rightsquigarrow v$ and $p \rightsquigarrow m$
	$v \neq m$	$v \neq m$	

Identity

(11) Hesperus is identical with Phosphorus.

- Violates the 1-st and 3-rd principle.

- Let v be Venus, and m is Mars.

-

'$h = p$'	i	j	
i	T	T	$h \rightsquigarrow v$ and $p \rightsquigarrow v$
j	F	F	$h \rightsquigarrow v$ and $p \rightsquigarrow m$
	$v \neq m$	$v \neq m$	

-

'$\dagger(h = p)$'	i	j	
i	T	F	$h \rightsquigarrow v$ and $p \rightsquigarrow v$
j	T	F	$h \rightsquigarrow v$ and $p \rightsquigarrow m$
	$v \neq m$	$v \neq m$	

Existence

(12) Aristotle does/did not exist.

- False, but contingent.

(13) Sherlock Holmes does/did not exist.

- Necessary, if true.
- "... since the domain of no possible world contains the actual person WE call Sherlock Holmes."

- Sentences with different status!

“Holmes Does Not Exist

‘Holmes does not exist’	i	j	k	
i	T	T	T	H does not refer
j	T	F	T	H denotes h
k	F	F	F	H denotes $SACD$

- This violates the first and the third principle again.

‘†(Holmes does not exist)’	i	j	k	
i	T	F	F	H does not refer
j	T	F	F	H denotes h
k	T	F	F	H denotes $SACD$

Some Motivating Examples

1. A dog entered the garden. It is barking.
   ?It is barking. A dog entered the garden.
2. If a cat is hungry it usually meows.
   ?It usually meows if a cat is hungry.
3. Rebecca married Thomas. She regrets that she married him.
   Rebecca regrets that she married Thomas. ?She married him.
4. Bob left. Conny started to cry. (Weak-hearted Conny!)
   Conny started to cry. Bob left. (Hard-hearted Bob!)
5. Max turned off the light. The room was pitch dark.
   ?The room was pitch dark. Max turned off the light.
6. I tell you your wife is cheating on you; now you know it.
   Now you know your wife is cheating on you; I tell you.
7. If Isabel is in the bathroom, Petra might be there, too.
   If Isabel is in the bathroom and nobody else is, Petra might be there, too.

Stalnaker's Truism and Veltman's Slogan

- "...acts of assertion affect, and are intended to affect, the context, in particular the attitudes of the participants in the situation."
- "...you know the meaning of a sentence if you know the change it brings about in the information state of anyone who accepts the news conveyed by it."
- $s[\![\bullet]\!]_{...}$ is an update function;
  - What is s, formally, intuitively?
  - What does $[\![\bullet]\!]$ do with it?

Update Semantics

- information states s are sets of *possibilities* i, which are valuations of atomic propositions

- $s[\![p]\!] = \{i \in s \mid p \in i\}$
  $s[\![\neg\phi]\!] = s \setminus s[\![\phi]\!]$
  $s[\![\phi \wedge \psi]\!] = s[\![\phi]\!][\![\psi]\!]$
  $s[\![\phi \rightarrow \psi]\!] = \{i \in s \mid \textit{if } i \in s[\![\phi]\!] \textit{ then } i \in s[\![\phi]\!][\![\psi]\!]\}$

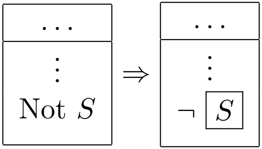
- $s[\![\Diamond\phi]\!] = \{i \in s \mid s[\![\phi]\!] \neq \emptyset\}$
  $s[\![\partial\phi]\!] = \{i \in s \mid s[\![\phi]\!] = s\}$

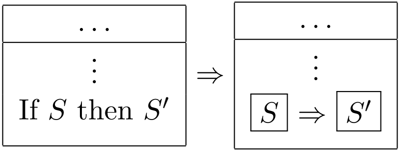
- non-commutative conjunction
- no identity axioms
- non-monotone entailment

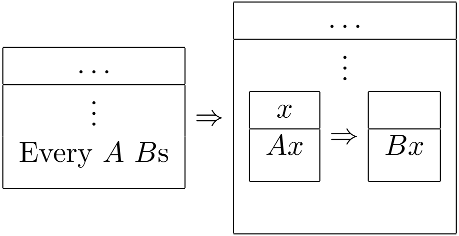
Discourse Representation Theory (1984/1993)

- information states K are structures *representing* the contents of a discourse under interpretation
- the 1984/1993 versions employ 'top down' construction
- $$\begin{array}{|c|}\hline x_1, \ldots, x_n \\ \hline \phi_1 \\ \vdots \\ \phi_m \\ \hline \end{array} \bigoplus S, S', \ldots \;\Rightarrow\; \begin{array}{|c|}\hline x_1, \ldots, x_n \\ \hline \phi_1 \\ \vdots \quad S \\ \phi_m \\ \hline \end{array} \bigoplus S', \ldots$$

Further Deconstruction (Schematic)

- 

- 

- 

Further Deconstruction (Schematic)

- $$\begin{array}{|c|}\hline x_1, \ldots, x_n \\ \hline \vdots \\ \text{Some } A\ B\text{s} \\ \hline \end{array} \Rightarrow \begin{array}{|c|}\hline x_1, \ldots, x_n, x_{n+1} \\ \hline \vdots \\ Ax_{n+1} \\ Bx_{n+1} \\ \hline \end{array}$$

- $$\begin{array}{|c|}\hline x_1, \ldots, x_n \\ \hline \vdots \\ \text{He } B\text{s} \\ \hline \end{array} \Rightarrow \begin{array}{|c|}\hline x_1, \ldots, x_n, x_{n+1} \\ \hline \vdots \\ x_i = x_{n+1} \\ Bx_{n+1} \\ \hline \end{array}$$