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INDIVIDUAL CONCEPTS IN MODAL PREDICATE LOGIC ⋆

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ABSTRACT. The article deals with the interpretation of propositional attitudes in the framework of modal predicate logic. The first part discusses the classical puzzles arising from the interplay between propositional attitudes, quantifiers and the notion of identity. After comparing different reactions to these puzzles it argues in favor of an analysis in which evaluations of *de re* attitudes may vary relative to the ways of identifying objects used in the context of use. The second part of the article gives this analysis a precise formalization from a model- and proof-theoretic perspective.

KEY WORDS: methods of cross-identification, pragmatics, propositional attitudes

1. INTRODUCTION

Suppose (1a) is true and (1b) is false. What is the truth value of (1c) under the assignment that maps the variable x to the individual which is Cicero and Tully?

(1a) Philip is unaware that Tully denounced Catiline.

(1b) Philip is unaware that Cicero denounced Catiline.

(1c) Philip is unaware that x denounced Catiline.

After 50 years, this question from Quine[1] is still puzzling logicians, linguists and philosophers. In the present article, this and other paradoxes of *de re* propositional attitude reports are discussed in the framework of modal predicate logic. In the first part of the article, I compare different reactions to these paradoxes and I argue in favor of an analysis in which evaluations of attitude reports are relativized to the ways of identifying objects used in the specific circumstances of an utterance. The insight that different methods of identification are available and can be used in different contexts is not new in the logical-philosophical literature[2] and it is also not without problems. In the second part of the article, I give this insight a precise formalization, which in the same go solves the associated problems, and sketch an analysis of the pragmatics of propositional attitudes in general.

⋆ This material has grown out of Chapters 2 and 4 of my PhD thesis 'Quantification under Conceptual Covers.'

2. SETTING THE STAGE: THE *de re–de dicto* DISTINCTION

The present article is about the interaction between propositional attitudes, quantifiers and the notion of identity. All of the conceptual difficulties arising from such interplay were first addressed by Quine, in his classic papers in the 50–60s.

Quine (1953) discusses cases of failure of the principle of *substitutivity of identicals*. According to this principle, originally formulated by Leibniz, co-referential expressions are interchangeable everywhere *salva veritate*. Quine considers the following example:

(2) 'Cicero' contains six letters.

Although Cicero is Tully, the substitution of the second for the first does not preserve the truth value of the sentence. The following is false:

(3) 'Tully' contains six letters.

The principle of substitutivity fails to hold in this case.

Another principle which clearly fails in connection with this kind of examples is the principle of *existential generalization*. If we apply such a principle to example (2) we obtain:

(4) $\exists x$('x' contains six letters)

which 'consists merely of a falsehood – namely "The 24th letter of the alphabet contains six letters." – preceded by an irrelevant quantifier' (Quine, 1953, p. 147). Contexts like quotations in which substitution of co-referential names may not preserve truth value are called *referentially opaque* by Quine. Given the difficulties illustrated by example (4), Quine has taken the view that quantification into opaque contexts is always misguided.

Propositional attitudes and modalities also create referential opaque contexts. Consider, for example, belief attributions, as in the examples in (5). The following three sentences are mutually consistent:

(5a) Philip believes that Cicero denounced Catiline.

(5b) Cicero is Tully.

(5c) Philip does not believe that Tully denounced Catiline.

Substitution of co-referential terms in belief contexts is not always allowed. Furthermore, consider sentence (6) which should be derivable from (5a) according to the principle of existential generalization. The problem with this sentence is that we cannot identify the object that according to Philip denounced Catiline. It cannot be Cicero, that is, Tully because to assume this would conflict with the truth of (5c).

(6) $\exists x$(Philip believes that x denounced Catiline)

Quine concludes that quantification in propositional attitude contexts is *always* unwarranted too, as in the quotation contexts considered above. But consider now the following sentence:

(7) Ralph believes that the president of Russia is bald.

Suppose Ralph wrongly believes that Jeltsin is the actual president of Russia, rather than Putin. How do we interpret (7)? Does it say that Ralph believes that Jeltsin is bald or Putin? Intuitively (7) can have both readings. On the first reading, the term 'the president of Russia' is presented as part of Ralph's belief and is interpreted from his perspective, so inside the scope of the belief operator. On the second reading, the same description is not taken to belong to Ralph's conceptual repertoire, but it is used to denote the actual president of Russia so that the description is interpreted from our perspective, thus outside the belief operator. Now consider another situation. Suppose Ralph does not have any idea about who the president of Russia is. Sentence (7) is again ambiguous. On the first reading, in which 'the president of Russia' is interpreted from Ralph's perspective, Ralph has an unspecific belief about whoever is the president of Russia; on the other reading, in which reference is made from our perspective, it is asserted that Ralph believes of Putin, who is *de facto* the president of Russia, that he is bald.

Now, if we assume the first reading of (7), substitution of co-referential terms can change the truth value of the sentence. Under this reading, (7) and (8) can have different truth values and the principle of substitutivity fails.

(8) Ralph believes that Putin is bald.

On the other hand, substitutivity is warranted, if we assume the second reading. If we interpret the relevant terms from the speaker's perspective, (7) is true iff (8) is true.

Furthermore, while existential generalization can intuitively not be warranted in the first case, it is always in the second case. Consider the second described situation. On the first interpretation of (7), the derivation of (9) is problematic, but it is intuitively justified on the second reading.

(9) $\exists x$(Ralph believes that x is bald)

From this example we can conclude that belief contexts are not always opaque. Quine's (1953) conclusion was too drastic after all. There are readings of belief reports for which the principle of substitutivity of identicals and existential generalization do hold. These have been called *de re* readings. Belief contexts which do not warrant these principles, and so are referentially opaque, are called *de dicto*.

The existence of unproblematic cases of quantification into propositional attitude contexts is recognized by Quine himself in (Quine, 1956), where he discusses the following classical example also illustrating the *de re–de dicto* contrast:

(10) Ralph believes that someone is a spy.
a. Ralph believes there are spies.
b. There is someone whom Ralph believes to be a spy.

Example (10) is ambiguous between a *de dicto* reading, paraphrased in (10a), asserting that Ralph believes that the set of spies is not empty, and a *de re* reading, paraphrased in (10b), saying that there is a particular individual to whom Ralph attributes espionage. 'The difference is vast: indeed, if Ralph is like most of us, (10a) is true and (10b) is false' (Quine, 1956, p. 178).

In the next section, I introduce modal predicate logic and I show how it can deal with *de re* and *de dicto* distinction.

3. Modal Predicate Logic

In this section I introduce possible world semantics for Modal Predicate Logic (MPL). The presentation is based on (Hughes and Cresswell, 1996).

A *language* $\mathcal{L}$ of modal predicate logic takes the following symbols as primitive:

(1) For each natural number $0 \leq n$ a (possibly finite but at most denumerably infinite) set $\mathcal{P}$ of n-place *predicates*.
(2) A (possibly finite but at most denumerably infinite) set $\mathcal{C}$ of *individual constants*.
(3) A denumerably infinite set $\mathcal{V}$ of *individual variables*.
(4) The symbols $\neg$, $\wedge$, $\exists$, $\Box$, $=$, (, and).

The following formation rules specify which expressions are to count as well formed of our language:

(R0) Any individual constant in $\mathcal{C}$ or variable in $\mathcal{V}$ is a term in $\mathcal{L}$.
(R1) Any sequence of symbols formed by an n-place predicate followed by n terms is a well formed formula (wff).
(R2) If t and t' are terms, then $t = t'$ is a wff.
(R3) If ϕ is a wff, then $\neg\phi$ is a wff.
(R4) If ϕ is a wff and x is a variable, then $\exists x\phi$ is a wff.
(R5) If ϕ and ψ are wffs, so is $(\phi \wedge \psi)$.
(R6) If ϕ is a wff, then $\Box\phi$ is a wff.
(R7) Nothing else is a wff.

We adopt the standard abbreviations $\phi \rightarrow \psi = \neg(\phi \wedge \neg\psi)$, $\forall x\phi = \neg\exists x\neg\phi$ and $\Diamond\phi = \neg\Box\neg\phi$.

A *model* M for $\mathcal{L}_{MPL}$ is a quadruple $\langle W, R, D, I\rangle$ in which W is a non-empty set of possible worlds; R is a relation on W, D is a non-empty set of individuals; and I is an interpretation function which assigns for each $w \in W$ an element $I_w(c)$ of D to each individual constant c in $\mathcal{C}$, and a subset $I_w(P)$ of D^n to each n-ary predicate P in $\mathcal{P}$.

Well-formed expressions in $\mathcal{L}$ are interpreted in models with respect to an assignment function $g \in D^{\mathcal{V}}$ and a world $w \in W$.

DEFINITION 1 (MPL-Interpretation of Terms).

(i) $[t]_{M,w,g} = g(t)$ if t is a variable.
(ii) $[t]_{M,w,g} = I_w(t)$ if t is a constant.

We define now a satisfaction relation $\models$, holding between a world w and a formula ϕ in a model M and relative to an assignment g, saying that ϕ is true in M and w with respect to g.

DEFINITION 2 (MPL-Interpretation of Formulas).

$$\begin{array}{rl}
M, w \models_g Pt_1, \ldots, t_n & \text{iff } \langle [t_1]_{M,w,g}, \ldots, [t_n]_{M,w,g}\rangle \in I_w(P) \\
M, w \models_g t_1 = t_2 & \text{iff } [t_1]_{M,w,g} = [t_2]_{M,w,g} \\
M, w \models_g \neg\phi & \text{iff not } M, w \models_g \phi \\
M, w \models_g \phi \wedge \psi & \text{iff } M, w \models_g \phi \text{ and } M, w \models_g \psi \\
M, w \models_g \exists x\phi & \text{iff } \exists d \in D : M, w \models_{g[x/d]} \phi \\
M, w \models_g \Box\phi & \text{iff } \forall w' : wRw' : M, w' \models_g \phi
\end{array}$$

A formula is valid in a model M iff it is true with respect to all assignments and all worlds in M. A formula is valid in MPL iff it is valid in all models.

DEFINITION 3 (MPL-Validity). Let $M = \langle W, R, D, I \rangle$ be a model for $\mathcal{L}$ and ϕ a wff of $\mathcal{L}$.

$$\begin{aligned} M \models_{MPL} \phi \quad &\text{iff} \quad \forall w \in W, \forall g \in D^{\mathcal{V}} : M, w \models_g \phi \\ \models_{MPL} \phi \quad &\text{iff} \quad \forall M : M \models_{MPL} \phi \end{aligned}$$

The idea of using modal predicate logic to represent the logic of propositional attitudes derives from (Hintikka, 1962). For simplicity, I will deal with cases in which one propositional attitude, namely belief, is attributed to one person only. The set of worlds w' accessible from w, $\{w' \in W \mid wRw'\}$, is seen as the belief state $Bel(w)$ of the relevant subject in w. $Bel(w)$ represents the set of the subject's doxastic alternatives in w, that is the set of possibilities that are compatible with her belief in that world. A sentence like 'a believes that ϕ' is translated as $\Box\phi$. $\Box\phi$ is true in w iff ϕ is true in all worlds accessible from w. This intuitively means that a subject a believes that ϕ is true iff in all possible worlds compatible with what a believes, it is the case that ϕ; and that a does not believe that ϕ is true iff in at least one world compatible with what a believes, it is not the case that ϕ.

Principles of a Logic of Belief. Kripke (1963) points out that different types of modal notions can be characterized by certain axiom schemes that constraint the accessibility relation. For example, consider the following scheme which may fail to hold only in a model in which some possibility is inaccessible from itself:[3]

T $\Box\phi \rightarrow \phi$

T is a plausible principle for certain interpretations of the modal operators, for instance, metaphysical necessity or knowledge. If something is necessary true, then it is true. And what is known must be the case. Thus a modal system which wants to capture the logic of these notions should only consider models with a reflexive accessibility relation ($\forall w : wRw$). On the other hand, **T** is not a plausible principle for a logic of belief. If you believe that it is raining, it does not follow that it is raining, because you might be wrong. Belief is not a factive notion, contrary to knowledge. Other schemes, however, are intuitively valid when it concerns beliefs. Belief is normally taken to satisfy positive and negative introspection. If you (do not) believe something, you believe that you (do not) believe it. The following two principles, thus, are plausible for a logic of belief.

4 $\Box\phi \rightarrow \Box\Box\phi$

E $\neg\Box\phi \rightarrow \Box\neg\Box\phi$

4 corresponds to the transitivity of R ($\forall w, w', w'' : wRw' \;\&\; w'Rw'' \Rightarrow wRw''$). **E** expresses the fact that R is a Euclidean relation ($\forall w, w', w'' : wRw' \;\&\; wRw'' \Rightarrow w'Rw''$). A further assumption which is often made (see, for instance, Hintikka, 1962) is that only consistent belief states are taken into consideration. If you believe ϕ, then ϕ is consistent with your belief state.

D $\Box\phi \rightarrow \Diamond\phi$

This principle is satisfied in all models in which each world has at least one accessible world ($\forall w : \exists w' : wRw'$). A relation which satisfies this condition is called a serial relation.

A last principle we should mention is **K** which is intuitively valid given any interpretation of $\Box$.

K $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$

It is easy to show that **K** holds in every MPL model, no matter what accessibility relation it has.

To summarize, in addition to **K**, the following three principles, corresponding to the respective conditions on the accessibility relation, will be adopted in what follows:

1. *Consistency* (Serial Relations)

   D $\Box\phi \rightarrow \Diamond\phi$

2. *Positive Introspection* (Transitive Relations)

   4 $\Box\phi \rightarrow \Box\Box\phi$

3. *Negative Introspection* (Euclidian Relations)

   E $\neg\Box\phi \rightarrow \Box\neg\Box\phi$

Reflexivity (Factivity) expressed by principle **T** and Symmetry expressed by the following principle **B**: $\Diamond\Box\phi \rightarrow \phi$ are not assumed. **B** is not a reasonable principle for belief. Not everything which is consistent to believe must be the case.

Belief and Quantification. We consider now how MPL deals with the interaction between quantifiers and the belief operator.

First of all the present semantics validates the following two schemes which are known as the Barcan Formula and its Converse:[4]

BF $\forall x\Box\phi \rightarrow \Box\forall x\phi$

CBF $\Box\forall x\phi \rightarrow \forall x\Box\phi$

Also the following related 'mixed' principle, which I call the Principle of Importation, holds in MPL:

IM $\exists x \Box \phi \rightarrow \Box \exists x \phi$

Its converse, however, which will be called the Principle of Exportation, is *not* generally valid:

EX $\Box \exists x \phi \rightarrow \exists x \Box \phi$

The failure of **EX** is crucial for the MPL representation of the *de re–de dicto* distinction.

De re and de dicto. In MPL, the *de re–de dicto* contrast can be expressed by means of permutation of components of formulae, as in (Russell, 1905). Sentences like (11) or (12) can be assigned the two logical forms in (a) and (b):

(11) Ralph believes that someone is a spy.

a. $\Box \exists x S(x)$

b. $\exists x \Box S(x)$

(12) Ralph believes that the president of Russia is bald.

a. $\Box \exists x (r = x \wedge B(x))$

b. $\exists x (r = x \wedge \Box B(x))$

The (a) logical forms express the *de dicto* readings of the sentences with possibly different individuals being spies or presidents in different doxastic alternatives of Ralph. The (b) logical forms express the *de re* readings, in which one and the same individual is ascribed espionage or baldness in all worlds compatible with what Ralph believes. The *de re–de dicto* distinction is represented in terms of a scope ambiguity. *De re* belief reports are sentences which contain some free variable in the scope of a belief operator. Note that the two logical forms $\Box \exists x (r = x \wedge B(x))$ and $\Box B(r)$ are equivalent in the present semantics. I will use the latter representation for the *de dicto* reading of sentences like (12) in the following discussion.

It is easy to see that by means of these representations, MPL manages to tackle the intuitions about the *de re* and the *de dicto* belief exposed in Section 2. Let us see first how the referential opacity of *de dicto* belief is accounted for.

MPL invalidates the following unrestricted versions of the principles of substitutivity of identicals and of existential generalization:

SI $t_1 = t_2 \rightarrow (\phi[t_1] \rightarrow \phi[t_2])$

EG $\phi[t] \rightarrow \exists x \phi[x]$

where $\phi[t_1]$ and $\phi[t_2]$ differ only in that the former contains the term t_1 in one or more places where the latter contains t_2. The two principles can fail in the presence of some belief operator when applied to arbitrary singular terms.[5] The reason for this is that the interpretation of a belief operator involves a shift of the world of evaluation and two terms can refer to one and the same individual in one world and yet fail to co-refer in some other. Given this fact, MPL can easily account for the consistency of the following three sentences, where (13a) and (15a) are assigned a *de dicto* interpretation:

(13a) Ralph believes that the president of Russia is bald.

(13b) $\Box B(r)$

(14a) The president of Russia is Putin.

(14b) $r = p$

(15a) Ralph does not believe that Putin is bald.

(15b) $\neg\Box B(p)$

Even if 'Putin' and 'the president of Russia' denote one and the same man in the actual world, thus making the identity in (14) true, they can refer to different men in the worlds conceived possible by Ralph. For this reason, (13) and (15) can both be true. The principle of substitutivity of identicals does not hold in general.[6] The substitutivity puzzle involving proper names, illustrated by the following examples, can be handled in the same way.

(16a) Philip believes that Cicero denounced Catiline.

(16b) $\Box\phi(c)$

(17a) Cicero is Tully.

(17b) $c = t$

(18a) Philip does not believe that Tully denounced Catiline.

(18b) $\neg\Box\phi(t)$

The intuitive consistence of these three sentences can be accounted for by assuming that in different doxastic alternatives a proper name can denote different individuals. The failure of substitutivity of co-referential terms (in particular proper names) in belief contexts does not depend on the ways in which terms actually refer to objects (so this analysis is not in opposition with Kripke's (1972) theory of proper names), it is simply due to the possibility that two terms that actually refer to one and the same individual are not believed by someone to do so.[7]

Similarly we can explain why we cannot always existentially quantify with respect to a term which occurs in a belief context. The term 'the president of Russia' may be such that it refers to different men in Ralph's doxastic alternatives (and in the actual world), and, therefore, the *de dicto* reading (19b) of a sentence like (19a) does not necessarily imply (20) or (21):

(19a) Ralph believes that the president of Russia is bald.

(19b) $\Box B(r)$

(19c) $\exists x(x = r \wedge \Box B(x))$

(20a) There is someone whom Ralph believes to be bald.

(20b) $\exists x \Box B(x)$

(21a) Ralph believes Putin to be bald.

(21b) $\exists x(x = p \wedge \Box B(x))$

On the other hand, both (20) and (21) are obviously derivable[8] from the *de re* reading (19c) of (19a) and, therefore, the referential transparency of *de re* belief is also accounted for.

The contrast between the *de re* and the *de dicto* logical forms is a genuine one. Existential generalization and exportation of terms from belief contexts is not generally allowed. In order for existential generalization (or term exportation) to be applicable to a term t occurring in the scope of a belief operator, t has to denote the same individual in all doxastic alternatives of the relevant agent (plus the actual world). The following two principles are valid in MPL, if we assume consistency, positive and negative introspection:[9]

$$\textbf{EG}_{\Box}\quad \exists x \Box x = t \rightarrow (\Box\phi[t] \rightarrow \exists x \Box\phi[x])$$

$$\textbf{TEX}_{\Box}\quad \exists x(x = t \wedge \Box x = t) \rightarrow (\Box\phi[t] \rightarrow \exists x(x = t \wedge \Box\phi[x]))$$

Sentences like $\exists x(x = t \wedge \Box x = t)$ are used by a number of authors, notably Hintikka, as representations of knowing-who constructions. $\text{TEX}_{\Box}$ says that a term t is exportable from a belief context if we have as an additional premise that the relevant subject knows who t is. In MPL, in order to have a *de re* belief we need to know who somebody is.

Double Vision Puzzles

Although MPL can account for the *de re–de dicto* distinction, its solution to the substitutivity paradox is not fully satisfactory. Since variables refer to one and the same individual in all possible worlds, the following version of the substitutivity principle holds in MPL:

SIv $x = y \rightarrow (\phi[x] \rightarrow \psi[y])$

As argued by Church (1982), substitutivity paradoxes can be constructed which depend on variables rather than descriptions or names. It is easy to see from **SIv** that MPL validates the following scheme:[10]

LIv $x = y \rightarrow \Box x = y$

Furthermore in serial frames, principle **CLNIv** follows as well:[11]

CLNIv $\Box x \neq y \rightarrow x \neq y$

Now consider the formulation of the two principles in ‘quasi-ordinary’ language:

(22) For every x and y, if $x = y$, then George IV believes that $x = y$.

(23) For every x and y, if George IV believes that $x \neq y$, then $x \neq y$.

Example (23) can be understood to say that one individual cannot be believed by George IV to be two. For example, George IV cannot fail to recognize as the same individual, an individual encountered on two different occasions. Example (24) says that George IV can make individuals distinct by merely believing that they are distinct. In MPL, George IV, as well as anyone with consistent beliefs, is predicted to have such incredible powers. These two predictions can intuitively be accepted ‘only on the doubtful assumption that belief properly applies “to the fulfillment of condition by objects” quite “apart from special ways of specifying” the objects’.[12] Following Church, I call this assumption the principle of *transparency of belief*. In the literature, a series of so called double vision situations have been discussed which illustrate the problematic nature of such a principle. In all of these examples, we find someone who knows an individual in different guises, without realizing that it is one and the same individual.

A famous case is the one discussed by Quine (1956). Quine tells of a man called Ralph, who ascribes contradictory properties to Ortcutt since, having met him on two quite different occasions, he is ‘acquainted’ with him in two different ways. Another well-known example is described in

(Kripke, 1979). In Kripke's story, the bilingual Pierre assents to 'Londres est jolie' and denies 'London is pretty', because he does not recognize that the ugly city where he lives now, which he calls 'London', is the same city as the one he calls 'Londres', about which he has heard while he was in France. In a third situation, described in (Richards, 1993), a man does not realize that the woman to whom he is speaking through the phone is the same woman he sees across the street and who he perceives to be in some danger. In such a situation the man might sincerely utter: 'I believe that she is in danger,' but not 'I believe that you are in danger', although the two pronouns 'she' and 'you' refer to one and the same woman. Let me expand upon the situation discussed in (Quine, 1956):

> There is a certain man in a brown hat whom Ralph has glimpsed several times under questionable circumstances on which we need not enter here; suffice it to say that Ralph suspects he is a spy. Also there is a grey-haired man, vaguely known to Ralph as rather a pillar of the community, whom Ralph is not aware of having seen except once at the beach. Now Ralph does not know it but the men are one and the same. (Quine, 1956, p. 179)

Consider the following sentence:

(24a) Ralph believes that x is a spy.

(24b) $\Box S(x)$

Is (24) true under an assignment which maps x to the individual Ortcutt which is the man seen on the beach *and* the man with the brown hat?[13] As Quine observes, the ordinary notion of belief seems to require that although (24) holds when x is specified in one way, namely as the man with the brown hat, it may yet fail when the same x is specified in some other way, namely as the man seen on the beach. Belief 'does not properly apply to the fulfillment of conditions by *objects* apart from special ways of specifying them' (Quine, 1953, p. 151). In standard modal predicate logic, we fail to capture this ordinary sense of belief. Since variables range over bare individuals, we cannot account for the fact that sentences like (24) depend on the way of specifying these individuals. This feature also implies that in MPL the following two sentences cannot both be true, unless we want to charge Ralph with contradictory beliefs:

(25a) Ralph believes Ortcutt to be a spy.

(25b) $\exists x(x = o \wedge \Box S(x))$

(26a) Ralph believes Ortcutt not to be a spy.

(26b) $\exists x(x = o \wedge \Box\neg S(x))$

In MPL we cannot avoid the inference from (25) and (26) to (27):

(27a) Ralph believes Ortcutt to be a spy and not to be a spy.

(27b) $\exists x(x = o \wedge \Box(S(x) \wedge \neg S(x)))$

Example (27) says that Ralph's belief state is contradictory. This prediction clashes with our intuitions about this case. On the one hand, since Ralph would assent to the sentence: 'That man with the brown hat is a spy', we are intuitively allowed to infer (25). On the other hand, since 'Ralph is ready enough to say, in all sincerity, 'Bernard J. Ortcutt is no spy',' (Quine, 1956, p. 179) we are also ready to infer (26). But this should not imply that Ralph has contradictory beliefs. Ralph is not 'logically insane', he simply lacks certain information.

In the following section, I will discuss three refinements of the standard modal predicate semantics which have been proposed as a response to Quine's intriguing puzzle, and I will show that they also raise problems of their own.

4. Contingent Identity Systems

Consider the property P which an object x has iff Ralph believes of x that x is a spy. From the discussion in the previous section, it seems that it depends on the way of referring to x whether P applies to x. A number of systems have been proposed that try to account for this dependence. Here I just consider what Hughes and Cresswell (1996) call Contingent Identity (CI) systems, based on the framework of modal predicate logic. In CI systems, the principles of necessary (non-)identity **LIv** and **LNIv** and their converses do not hold. This result is obtained by allowing a variable to take different values in different worlds. A standard way to do this is to let variables range over so-called individual concepts.[14] An *individual concept* is a total function from possible worlds in W to individuals in D.

4.1. *Quantifying over All Concepts*

In the first Contingent Identity semantics we will consider (CIA), variables are taken to range over *all* individual concepts in $IC = D^W$. Language and models are defined as in MPL. Well-formed expressions are interpreted in models with respect to a world and an assignment function $g \in IC^{\mathcal{V}}$. Variables are crucially assigned concepts in IC, rather than individuals in D. The denotation of a variable x with respect to an assignment g and a world w is the instantiation $g(x)(w)$ of $g(x)$ in w.

DEFINITION 4 (CI-interpretation of variables). $[x]_{M,w,g} = g(x)(w)$

In the semantics, we only have to adjust the clause dealing with existential quantification.

DEFINITION 5 (Quantification over all individual concepts).

$$M, w \models_g \exists x\phi \quad \text{iff} \quad \exists c \in IC : M, w \models_{g[x/c]} \phi$$

It is easy to see that CIA does not validate **LIv** and **LNIv** and their converses. Let $M = \langle W, R, D, I \rangle$ be such that $W = \{w, w'\}$ and $R = \{\langle w, w' \rangle, \langle w', w' \rangle\}$; and let $g, g' \in IC^{\mathcal{V}}$ be such that $g(x)(w) = g(y)(w)$ and $g(x)(w') \neq g(y)(w')$, and $g'(x)(w) \neq g'(y)(w)$ and $g'(x)(w') = g'(y)(w')$ respectively. Then we have:

(i) $M, w, g \not\models_{CIA} x = y \rightarrow \Box x = y$;
(ii) $M, w, g \not\models_{CIA} \Box x \neq y \rightarrow x \neq y$;
(iii) $M, w, g' \not\models_{CIA} x \neq y \rightarrow \Box x \neq y$;
(iv) $M, w, g' \not\models_{CIA} \Box x = y \rightarrow x = y$.

By invalidating **LIv**, CIA avoids the double vision paradoxes. Given the situation described by Quine, a sentence like (24) is true under an assignment which maps x to the concept λw[the man with the brown hat]$_w$ and false under an assignment which maps x to the concept λw[the man seen on the beach]$_w$.

(24a) Ralph believes that x is a spy.

(24b) $\Box S(x)$

In this way the dependence of belief on the ways of specifying the intended objects is accounted for. Furthermore, (25) and (26) below do not entail the problematic (27):

(25) $\exists x(x = o \wedge \Box S(x))$

(26) $\exists x(x = o \wedge \Box \neg S(x))$

(27) $\exists x(x = o \wedge \Box(S(x) \wedge \neg S(x)))$

We can infer (28), which, as it is easy to see, does not entail that Ralph's beliefs are inconsistent:

(28) $\exists x(x = o \wedge \exists y(o = y \wedge \Box(S(x) \wedge \neg S(y))))$

The analysis of *de re* belief reports formalized by CIA can be intuitively formulated as follows:

> The sentence 'a believes b to be ϕ' is true iff there is a representation α such that α is actually b and a would assent to 'α is ϕ'.

To believe *de re* of x that x has P is to ascribe P to x under some representation.

Although in a different framework,[15] Quine (1956) predicts similar truth conditions for *de re* belief attributions. In order to account for ordinary cases of quantification into belief contexts, Quine distinguishes two notions of belief, *notional* (belief_1) and *relational* (belief_2). The latter contains one or more of the crucial terms in a purely referential position and, therefore, sustains both substitution of identicals and existential generalization. For example, a sentence like:

(29) Ralph believes that Ortcutt is a spy.

is assigned two interpretations, represented as follows:

(30) Ralph believes_1 ('Ortcutt is a spy')

(31) Ralph believes_2('x is a spy', Ortcutt)

According to Quine, the relational interpretation (31), which corresponds to the *de re* reading, is implied[16] by any sentence of the form:

(32) Ralph believes_1 ('α is a spy')

together with the simple identity '$\alpha =$ Ortcutt'.

As noticed by Kaplan (1969), there is a problem with this analysis. Upon a closer inspection, Quine's account (or CIA more in general) fails to capture the intuitions that originally led us to a distinction between the *de re* and *de dicto* representations. The shortest spy problem illustrates why.

The Shortest Spy

It is easy to see that CIA validates the principle of exportation:

$$\textbf{EX} \quad \Box\exists x\phi \rightarrow \exists x\Box\phi$$

It follows that the general form of existential generalization **EG** and term exportation from belief contexts are also validated:

$$\textbf{TEX} \quad \Box\phi[a] \rightarrow \exists x(x = a \wedge \Box\phi[x])$$

The following two examples illustrate why the validity of these schemes clashes with our intuitions about *de re* belief.

Consider the following case discussed in (Kaplan, 1969). Suppose Ralph believes there are spies, but does not believe of anyone in particular that she is a spy. He further believes that no two spies have the same height which entails that there is a shortest spy. In such a situation, the *de re* reading of Quine's spy example (33), which 'was originally intended to express a fact that would interest the F.B.I.' (Kaplan, 1969, p. 220), is intuitively false.

(33) There is someone whom Ralph believes to be a spy.

The problem of the present semantics is that such a reading is not captured by the following representation:

(34) $\exists x \Box S(x)$

Given the circumstances described above, we have no troubles in finding a value for x under which $\Box S(x)$ is true, namely the concept λw[the shortest spy]$_w$, therefore, (34) is true in CIA and hence cannot be used to express (33). The classical representation of the *de re–de dicto* contrast in terms of scope permutation is no longer available. The following example illustrates the same difficulty. Consider the *de re* sentence:

(35) Ralph believes Putin to be the president of Russia.

Example (35) is intuitively false in a situation in which Ralph believes that Jeltsin is the president of Russia. In CIA, however, the standard *de re* representation of (35), namely (36) is implied by any sentence of the form (37) together with the simple identity $\alpha = p$.

(36) $\exists x(x = p \wedge \Box x = r)$

(37a) Ralph believes that α is the president of Russia.

(37b) $\Box\, \alpha = r$

In particular, it is implied by a trivially true sentence like (38) in case Putin is *de facto* the president of Russia.

(38a) Ralph believes that the president of Russia is the president of Russia.

(38b) $\Box\, r = r$

Thus, in CIA, (36) is true in the described situation and, therefore, cannot serve as a representation (35).

The validity of (term) exportation conflicts with our intuitions about the significant difference between the *de re* and *de dicto* readings of belief attributions. One conclusion we could draw from these examples is the inadequacy of an analysis of *de re* belief which assumes that they involve quantification over concepts, rather than objects. Principles **EX** and **TEX** are indeed very natural, if we take quantifiers to range over concepts. If *a* believes that someone is the so and so, then there is a concept, viz. the so and so, such that *a* believes that it is the so and so. On the other hand, if we assumed, instead, that *de re* belief applies to bare individuals rather than concepts, we would go back to MPL with its double vision difficulties. Many authors have therefore maintained that an analysis of *de re* belief involving quantification over ways of specifying individuals is on the right track. What is needed, if we want to solve the difficulties presented in the present section, is not a return to quantification over individuals rather than representations, but 'a frankly inequalitarian attitude'[17] towards these representations. This is Quine's diagnosis of the 'shortest spy cases', further developed by Kaplan (1969).[18] According to this view, *de re* belief attributions do involve quantifications over representations, yet not over all representations. 'The shortest spy' or 'the president of Russia' in the examples above are typical instances of representations that should not be allowed in our domain of quantification. In the next subsection, I present and investigate a second contingent identity system which formally works out this strategy. It will turn out, however, that also this kind of analysis is not fully satisfactory.

4.2. *Quantifying over Suitable Concepts*

Standard MPL was too stringent in allowing only plain individuals as possible values for our variables, and CIA was too liberal in allowing all concepts to count as 'objects'. An adequate semantics might be one which allows some (possibly non-rigid) concepts to count as possible values for our variables, but not all. Such a semantics could be obtained by taking models which specify which sets of concepts are to count as domains of quantification. These *models* will be quintuples $\langle W, R, D, S, I\rangle$ in which W, R, D, I are as above and $S \subseteq IC$. In this second Contingent Identity semantics CIB, variables are taken to range over a subset of the set of all concepts. Assignments $g \in S^{\mathcal{V}}$ map individual variables to elements of S.

DEFINITION 6 (Quantification over suitable concepts).

$$M, w \models_g \exists x\phi \quad \text{iff} \quad \exists c \in S : M, w \models_{g[x/c]} \phi$$

It is easy to see that the notion of validity defined by CIB is weaker than the notion of MPL and of CIA validity. Indeed, all MPL and CIA models are CIB models, properly understood. CIB-validity thus entails MPL- and CIA-validity, but not the other way around.

PROPOSITION 1. *Let ϕ be a wff of $\mathcal{L}$.*

(i) $\models_{CIB} \phi \quad \Rightarrow \quad \models_{MPL} \phi$
(ii) $\models_{CIB} \phi \quad \Rightarrow \quad \models_{CIA} \phi$

Proof. The proof relies on the fact that for each MPL or CIA model $M = \langle W, R, D, I\rangle$ we can build two corresponding CIB models, M_{MPL} and M_{CIA}, which satisfy conditions (a) and (b) respectively, for all wffs ϕ:

(a) $M_{MPL} \models_{CIB} \phi \quad \Leftrightarrow \quad M \models_{MPL} \phi$
(b) $M_{CIA} \models_{CIB} \phi \quad \Leftrightarrow \quad M \models_{CIA} \phi$

The construction of the two models is straightforward. Let $M = \langle W, R, D, I\rangle$ be an MPL (or CIA) model. We build $M_{MPL} = \langle W', R', D', S_{MPL}, I'\rangle$ and $M_{CIA} = \langle W', R', D', S_{CIA}, I'\rangle$ as follows. We let W', R', D', I' be like W, R, D, I and S_{MPL} and S_{CIA} contain all and only rigid concepts and all concepts respectively: $S_{MPL} = \{\lambda w[d] \mid d \in D\}$ and $S_{CIA} = IC$. It is an easy exercise to see that conditions (a) and (b) are satisfied. But then for X ranging over MPL and CIA, $\not\models_X \phi$ implies for some M, $M \not\models_X \phi$ which implies for the corresponding model M_X, $M_X \not\models_{CIB} \phi$, which means $\not\models_{CIB} \phi$. □

Note that MPL-validity does not entail CIA-validity or the other way around:

(iii) $\models_{MPL} \phi \quad \not\Rightarrow \quad \models_{CIA} \phi$
(iv) $\models_{CIA} \phi \quad \not\Rightarrow \quad \models_{MPL} \phi$

Together with Proposition 1, this implies that CIB-validity is strictly weaker than MPL- and CIA-validity.

(v) $\models_{MPL} \phi \quad \not\Leftarrow \quad \models_{CIB} \phi$
(vi) $\models_{CIA} \phi \quad \not\Leftarrow \quad \models_{CIB} \phi$

The CIB semantics is very promising. By Proposition 1, clause (i), CIB does not validate **EX** or **TEX**, since they are not valid in MPL. Therefore, it seems to avoid the problem of the shortest spy. On the other hand, by Proposition 1, clause (ii), **LIv** is also invalidated, since it fails to hold in CIA, and therefore the double vision difficulties do not arise.

In CIB, *de re* belief attributions are analyzed as follows:

> The sentence '*a* believes *b* to be ϕ' is true iff there is a *suitable* representation α such that α is actually *b* and *a* would assent to 'α is ϕ'.

To believe *de re* of *x* that *x* has *P* is to ascribe *P* to *x* under some suitable representation α.

Although it uses a different framework,[19] the influential analysis in (Kaplan, 1969) can be classified in this group. In that article, Kaplan attempts a concrete characterization of the notion of a suitable representation with respect to an agent and an object. A necessary and sufficient condition for the truth of a sentence of the form 'a believes x to be P', is the existence of a representation α in the conceptual repertoire of the agent a such that (i) α denotes x, (ii) α is a name *of* x for a, (iii) α is sufficiently vivid, and (iv) a believes α is P.

So, for example, the following sentence is accepted iff there is a vivid name α of Putin in Ralph's conceptual repertoire such that Ralph believes that α is the president of Russia.

(36a) Ralph believes Putin to be the president of Russia.

(36b) $\exists x(x = p \wedge \Box x = r)$

I will not discuss Kaplan's analysis in detail, but just note that *de re* belief reports are analyzed as describing mental acts or states of the agent. Their truth or falsity depends on a fact about the belief state as such, and this is in accordance with a kind of semantics like CIB in which the set of suitable representations is selected by the model, rather than by a contextual parameter, since the model also fully determines the belief state of the one relevant subject.[20]

In CIB, the problematic exportation steps in the shortest spy and in the president examples are blocked simply by assuming that S does not contain the concepts λw[the shortest spy]$_w$ or λw[the president of Russia]$_w$, since the two descriptions are clearly not vivid names of the intended objects for our Ralph in the described circumstances. However, the examples below show that there still are problems with this type of analysis.

In CIB, the existence of a suitable representation α of an object b for a subject a such that a would assent to 'α is ϕ' is a *necessary* and a *sufficient* condition for the truth of the *de re* sentence 'a believes b to be ϕ'. I call this condition condition (A). In what follows I will discuss a number of examples which show that this bi-implication leads to a number of empirical difficulties. In the case of Odette's lover below (and that of Susan's mother), we find a counterexample to the necessity of condition (A). The following two examples (the theater and Ortcutt again) show condition (A) not to be sufficient either. This double failure finds a natural explanation once we recognize the context dependence of the notion of a suitable representation. On different occasions, different sets of representations can count as suitable depending on the circumstances of the conversation, rather than on the mental state of the relevant subject. The problem of CIB is that models encode the information about what are the suitable representations and, therefore, they are not equipped to account for this variability.

Odette's Lover. Consider the following situation described by Andrea Bonomi.

Thanks to some clues, Swann has come to the conclusion that his wife Odette has a lover, but he has no idea who his rival is, although some positive proof has convinced him that this person is going to leave Paris with Odette. So he decides to kill his wife's lover, and he confides his plan to his best friend, Theo. In particular, he tells Theo that the killing will take place the following day, since he knows that Odette has a rendezvous with her lover. [...] Unknown to Swann, Odette's lover is Forcheville, the chief of the army, and Theo is a member of the security staff which must protect Forcheville. During a meeting of this staff to draw up a list of all the persons to keep under surveillance, Theo (who, unlike Swann, knows all the relevant details of the story) says:

(39) Swann wants to kill the chief of the army.

meaning by this that Swann is to be included in the list. The head of the security staff accepts Theo's advice. [...] Swann is kept under surveillance. A murder is avoided. (Bonomi, 1995, pp. 167–168)

Let's see whether the system CIB can account for this example. On its *de dicto* reading, (39) is false for obvious reasons. On its *de re* reading, it is true only if we can find a relevant α among the suitable representations or, in Kaplan's terminology, among the vivid names for Swann of Forcheville, such that Swann wants to kill α. The only possible candidate in the described situation is the description 'Odette's lover'. Formally, the sentence is true only if the concept $\lambda w[\text{Odette's lover}]_w$ is in S. Now according to the most intuitive characterization of the notion of a vivid name of an object for a subject, the relevant concept should not count as a suitable one. Swann does not know who Odette's lover is. Condition (A) is not satisfied, and, therefore, CIB cannot account for the truth of Theo's report. In order to avoid this counterintuitive result a proponent of CIB could argue in favor of a weaker notion of a suitable representation. However, if our notion of a suitable representation were as weak as to tackle this example, it would be too weak to solve the problem of the shortest spy. If in order to account for the truth of (39) we say that 'Odette's lover' is a vivid name of Forcheville for Swann, then the following sentences would also be true in the described situation:

(40) Swann believes of the chief of the army that he is Odette's lover.

which seems to be unacceptable, intuitively. To summarize, either 'Odette's lover' counts as a suitable description of Forchevilles for Swann, or it does not. If it does not, condition (A) is not satisfied for either example and so CIB fails to account for the truth of (39). If it does, condition (A) is satisfied for both examples and, therefore, CIB fails to account for the unacceptability of (40). A natural way out of this impasse would be to accept that one and the same representation can be suitable in one occasion and not in another. But if the set of suitable descriptions is determined by the model as in CIB, this solution is not available.

In the following example, due to van Fraassen, we find another case illustrating the same point.

Susan's Mother

Susan's mother is a successful artist. Susan goes to college, where she discusses with the registrar the impact of the raise in tuition on her personal finances. She reports to her mother 'He said that I should ask for a larger allowance from home'. Susan's mother exclaims: 'He must think I am rich!' Susan, looking puzzled, says 'I don't think he has any idea who you are'. (van Fraassen, 1979, pp. 371–372)

van Fraassen analyzes the example as follows:

The information the mother intends to convey is that the registrar believes that Susan's mother is rich, while Susan misunderstands her as saying that the registrar thinks that such and such successful artist is rich. The misunderstanding disappears if the mother gives information about herself, that is, about what she had in mind. She relied, it seems, on the auxiliary assertion 'I am your mother'. (van Fraassen, 1979, p. 372)

I repeat the crucial sentence used by Susan's mother:

(41) He must think I am rich.

This utterance may be not fully felicitous, because it is ambiguous, but it is not false; indeed, Susan accepts the sentence after the clarification of her mother. Again a proponent of CIB faces a dilemma: either (a) he does not accept 'Susan's mother' as a vivid representation of the referent of the pronoun 'I' in (41) for the registrar or (b) he does. If (a), then CIB fails to account for the intuitive acceptability of (41); if (b), it fails to account for the unacceptability of the following sentence when used by Susan's mother in the same situation:

(42) He must think I am your mother.

Again, CIB has difficulties in explaining the difference in acceptability between the two utterances, because it has no explanation of why one and the same representation, namely 'Susan's mother', can be used on one occasion and not on the other.

What the examples of Odette's lover and of Susan's mother show is that the cognitive relation between the subject (Swann or the registrar) and the object of belief (Forcheville or the mother) does not always play an essential role in deciding about the acceptability of *de re* sentences. In both examples, a term t is exported even if the subject does not have any intuitively acceptable answer to the question 'Who is t?' Whether a representation is suitable or not depends in the two cases on what properties are ascribed under such a representation, that is, on a fact about the conversation rather than on Swann's or the registrar's belief state. An approach upon which the information about which concepts are suitable is stored in the model fails to account for such dependencies.

In the following two cases, we find counterexamples to the sufficiency of condition (A) and further illustrations of the evident context sensitivity of *de re* constructions.

At the Theater. Consider the following situation described in the 1999 edition of (Bonomi, 1983). Suppose Leo correctly believes that Ugo is the only one in town who has a bush jacket. Leo further believes that Ugo has climbed the Cervino mountain. One night Ugo lends his bush jacket to another friend of Leo, Pio. Wearing it, Pio goes to the theater. Leo sees him and believes he is Ugo and utters:

(43) That person has climbed the Cervino.

From then on, nothing happens to modify Leo's belief about the climbing abilities of his friends. By the way, he is informed of the well-known fact that Pio hates climbing and he would never say that Pio climbed the Cervino. Now consider sentence (44) uttered by a fourth person Teo in two different circumstances α and β.

(α) Two months later.

(β) The same evening at the theater immediately after Leo's utterance of (43).

(44) Leo believes that Pio has climbed the Cervino.

Bonomi observes a contrast between the acceptability of the sentence in the two contexts. In α, the sentence is hardly acceptable, although, as it seems, condition (A) is satisfied,[21] and so constitutes a counterexample to the sufficiency of such condition.

On the other hand, our intuitions about the acceptability of (44) in context β are less sharp and the sentence might be acceptable if uttered immediately after Leo's manifestation of his beliefs. In the following enriched context γ, the acceptability of (44) may be more evident:

(γ) After uttering (43), Leo goes to Pio and congratulates him for his great performance. Ugo, a fourth friend, asks Teo for an explanation of Leo's surprising behaviour.

It seems to me that in γ, (44) can be accepted as an appropriate answer to Ugo's question. All of the considered analyses have difficulties in explaining this kind of context relativity. For all of them, the *de re* reading of (44) is an eternal proposition whose truth value depends on Leo's belief state, which has not changed with respect to the relevant facts. So, sentences like (44) are either true or false, irrespective of the context in which they are uttered. CIA would predict that (44) is true since we have a description under which Leo believes of Pio that he climbed the mountain. A proponent of MPL or CIB would have to decide whether the perceptive contact between Pio and Leo at the theater counts as an acquaintance relation. If it does, then we have a suitable representation of Pio for Leo under which Leo makes the relevant attribution and this is a sufficient condition for the truth of (44) also in context α. If it does not, then by the necessity of condition (A) we fail to account for the appropriateness of (44) in (γ).

As a last example, consider the following variation on Quine's double vision puzzle, which constitutes a further illustration of the context relativity of *de re* belief reports.

Ortcutt Again. You can tell each half of the Ortcutt story separately. In one half Ralph sees Ortcutt wearing the brown hat. In the other he sees him on the beach. From the first story you can reason as in (45). From the second story as in (46).

(45a) Ralph believes that the man with the brown hat is a spy.
(45b) The man with the brown hat is Ortcutt.
(45c) So Ralph believes of Ortcutt that he is a spy.

(46a) Ralph believes that the man seen on the beach is not a spy.
(46b) The man seen on the beach is Ortcutt.
(46c) So Ralph does *not* believe of Ortcutt that he is a spy.

Although we do not have to assume that there is any change in Ralph's belief state, it seems unproblematic to say that Ralph believes Ortcutt to be a spy and Ralph does *not* believe Ortcutt to be a spy, depending on which part of the story you are taking into consideration. The challenge is how to account for the compatibility of (45c) and (46c). Their natural representations (47) and (48) respectively are obviously contradictory:

(47) $\exists x(x = o \wedge \Box S(x))$
(48) $\neg\exists x(x = o \wedge \Box S(x))$

Proponents of CIB (or CIA) could then argue that a correct representation for (46c) is not (48), but rather (49) which, in CIB, is not in contradiction with (47):

(49) $\exists x(x = o \wedge \neg\Box S(x))$

Contrary to MPL, in which the two logical forms (48) and (49) are equivalent, CIA and CIB predict a structural ambiguity for sentences like ‘Ralph does *not* believe Ortcutt to be a spy’, with a wide scope reading asserting that Ralph does not ascribe espionage to Ortcutt under any (suitable) representation, and a narrow scope reading asserting that there is a (suitable) representation under which Ralph does not ascribe espionage to Ortcutt. This ambiguity is automatically generated by any system which assumes that *de re* belief reports involve quantification over representations of objects rather than over the objects themselves. Intuitively, however, it is hard to detect an ambiguity in the natural language sentences.[22] Furthermore, such an account of the possible consistency of (45c) and (46c), would lack an explanation of the influence of the previous discourse on the acceptability of one *or* the other sentence. Intuitively (45c) is acceptable in (45), but not in (46) because the relevant description, namely ‘the man with the brown hat’, which is explicitly mentioned in (45), is absent in (46) and not salient in that context. Again CIB fails to account for this type of context sensitivity.

To summarize, in the first two cases we have seen how one and the same description (‘Odette’s lover’ and ‘Susan’s mother’) can or cannot be a suitable representation of an intended object for a subject whose beliefs are described, this depending on the circumstances of the utterance and the property ascribed. In the last two cases, we have seen one and the same *de re* belief report (examples (44) and (45c)) which obtains different truth values when it is used in different circumstances without any relevant change in the belief state of the subject. The system CIB, which assumes that *de re* belief attributions involve quantification over a set of suitable concepts determined by the model of interpretation, cannot account for any of these cases without automatically generating other problems.

From the examples discussed in this section we can conclude that although the problem of interpreting quantification into the scope of a belief operator can be seen as the problem of distinguishing suitable representations from non-suitable ones, it is not the cognitive relation between the subject of belief and the intended object alone, that can supply the central notion for this distinction. Rather it seems that other elements play a crucial role, namely the conversational circumstances in which the belief report is made, the property ascribed, and the interests and goals of the participants in the conversation. The pragmatic analysis in the following section tries to take into account such dependencies. In Section 5 it is worked out more systematically.

4.3. *Pragmatic Analysis*

In the previous section, I discussed a number of examples illustrating the dependence of *de re* belief on various pragmatic factors.[23]

Those examples suggest the following preliminary rough analysis of *de re* belief reports:

> The sentence 'a believes b to be ϕ' used in context C is true iff there is a representation α *suitable in C* such that α is actually b and a would assent to 'α is ϕ'.

Formally, a model is defined as in standard MPL and assignments are defined as in CIA. The interpretation function is relativized to a pragmatic parameter which selects sets of contextually salient concepts out of $IC = D^W$. Let Z be a set of concepts whose value is pragmatically supplied.

DEFINITION 7 (Quantification over contextually selected concepts).

$$M, w, Z \models_g \exists x\phi \quad \text{iff} \quad \exists c \in Z : M, w, Z \models_{g[x/c]} \phi$$

The idea behind the pragmatic analysis is that *de re* belief reports express different contents in different contexts, in the same way (or in a similar way) as sentences containing indexical expressions. In different circumstances, different sets of concepts are assumed to supply the domain of quantification of our quantifiers.[24] The interpretation of *de re* belief reports, which directly depends on how objects are identified across the boundary of different possible worlds, is crucially affected by this variability.

The analysis I propose in Section 5 is among the pragmatic approaches. It has however the following extra feature that I believe is not trivial and is not a matter of detail. It is assumed that not all sets of concepts can be pragmatically selected as domains of quantification, but only those satisfying two natural conditions. The first condition is that for each individual d in the domain and each world w, the selected set Z must contain a concept which identifies d in w. The second condition is that Z cannot contain overlapping concepts, i.e. concepts standing for one and the same individual in one world and for two different individuals in another. I call the former the *existence* condition and the latter the *uniqueness* condition. In the remaining part of the present section I present a number of arguments in favor of their assumption.

The question whether an individual can fail to be identifiable in Z in some world (*existence*) is equivalent to the question whether existential generalization can fail, if applied to sentences which do not contain any belief operator:

EG1 $\phi[t] \rightarrow \exists x\phi[x]$ (if ϕ is non-modal).

We expect principle **EG1** to hold in our semantics. Contrast the following two examples:

(50a) If Ralph believes that the president of Russia is a spy, then there is someone whom Ralph believes to be a spy.

(50b) $\Box S(r) \rightarrow \exists x \Box S(x)$

(51a) If the president of Russia is a spy, then there is someone who is a spy.

(51b) $S(r) \rightarrow \exists x S(x)$

While, as we have seen, (50) can intuitively fail to be generally valid, (51) cannot. The failure of existential generalization is a peculiarity of opaque contexts. Existential generalization intuitively holds in the absence of belief operators. Assuming the existence condition for our quantificational domains is a natural way of accounting for this intuition. Note that CIB, which, as I presented it, does not assume such condition, fails to validate **EG1**.[25]

The question whether we should allow overlapping concepts (*uniqueness*) is equivalent to the question whether the following principles can fail to be true assuming that x and y range over *one and the same* quantificational domain Z:

(52) $\forall x \forall y(x = y \rightarrow \Box x = y)$

(53) $\forall x \forall y(x \neq y \rightarrow \Box x \neq y)$

Intuitively, if x and y stand for individuals we expect the principles to hold, if they stand for representations of individuals we expect them to fail. Individuals do not split or merge when we move from one world to the other, whereas a characteristic property of representations is that two representations can coincide in one situation (denote one and the same individual) and split or not in another. Now, recall the principles of exportation **EX**: $\Box\exists x\phi \rightarrow \exists x\Box\phi$ and of term exportation **TEX**: $\Box\phi[a] \rightarrow \exists x(x = a \wedge \Box\phi[x])$. As we have already observed, if we take variables to range over representations, rather than individuals, then **EX** or **TEX** are intuitively plausible, but then instances of their conclusions cannot be used to express *de re* readings. It seems fair to conclude that *de re* attitudes are not about representations. By adopting uniqueness, we can capture this intuition. A concept in a set satisfying uniqueness cannot split or merge. To this extend it resembles an individual rather than a representation.

De re attitudes are not about representations. They are not about bare individuals either. Consider again Quine's double vision situation. We expect our semantics to account for the fact that the following sentences do not imply that Ralph has inconsistent beliefs:

(54a) Ralph believes Ortcutt to be a spy.

(54b) $\exists y(y = o \wedge \Box S(y))$

(55a) Ralph believes Ortcutt not to be a spy.

(55b) $\exists x(x = o \wedge \Box\neg S(x))$

The CI solution to this puzzle crucially involves the presence in the quantificational domain of two overlapping concepts, namely ‘the man on the beach’ and ‘the man with the brown hat’ which stand for one and the same individual in one world and for two different individuals in another. If we rule out overlapping concepts altogether it is not immediately clear how we can account for this case. On the one hand, *de re* belief reports are not about ways of specifying individuals, as I have argued above. On the other, their interpretation crucially depends on these identification methods, as illustrated by the Ortcutt case. The problem is how to combine these two intuitions. MPL accounts for the first intuition, but fails to capture the double vision cases. The CI systems’ solution of the double vision puzzles, leads directly to the problems discussed above. A pragmatic approach supplies us with a natural way out from this impasse. The compatibility of the two sentences (54b) and (55b) is captured by letting the variables x and y range over different sets of concepts. The availability of different sets of non-overlapping concepts as possible domains of quantification on different occasions, enables us to account for the dependence of belief reports on the ways of referring to objects (and so for double vision cases), without dropping the uniqueness condition, and so avoiding the counterintuitive results arising when we quantify over concepts rather than objects.

The conclusion that can be drawn from this discussion is that *de re* belief reports neither involve quantification over bare individuals (MPL), nor over ways of specifying these individuals (CI), but rather over individuals specified in one determinate way.

5. Conceptual Covers in Modal Predicate Logic

In this section, I present Modal Predicate Logic under Conceptual Covers (CC). In Section 5.1, I define the notion of a conceptual cover. Section 5.2 presents the semantics of CC. Section 5.3 discusses a number of applications and sketches an analysis of the pragmatics of attitude reports. Section 5.4 compares CC validity with classical MPL validity. Finally, Section 5.5 introduces an axiom system which provides a sound and complete characterization of the set of CC-valid formulas.

5.1. *Conceptual Covers*

A conceptual cover is a set of individual concepts that satisfies the following condition: in a conceptual cover, in each world, each individual constitutes the instantiation of one and only one concept.

Given a set of possible worlds W and a universe of individuals D, a *conceptual cover CC* based on (W, D) is a set of functions $W \rightarrow D$ such that:

$$\forall w \in W : \forall d \in D : \exists! c \in CC : c(w) = d$$

Conceptual covers are sets of concepts which exhaustively and exclusively cover the domain of individuals. In a conceptual cover each individual d is identified by at least one concept in each world (*existence*), but in no world is an individual counted more than once (*uniqueness*).

It is easy to prove that each conceptual cover and the domain of individuals have the same cardinality. In a conceptual cover, each individual is identified by one and only one concept. Different covers constitute different ways of conceiving one and the same domain.

Illustration. Consider the following situation. In front of Ralph stand two women. Suppose Ralph believes that the woman on the left, who is smiling, is Bea; and the woman on the right, who is frowning, is Ann. As a matter of fact, exactly the opposite is the case. Bea is frowning on the right and Ann is smiling on the left. In order to formalize this situation, we just need to distinguish two possibilities. The simple MPL model $\langle W, R, D, I \rangle$ visualized by the following diagram will suffice:

$$\begin{array}{lll} w_1 \mapsto & (\ddot\smile) & (\ddot\frown) \\ & [ann] & [bea] \\ & & \\ w_2 \mapsto & (\ddot\smile) & (\ddot\frown) \\ & [bea] & [ann] \end{array}$$

W consists of two worlds w_1 and w_2. w_2 is the only world accessible from w_1 for Ralph. D consists of two individuals $(\ddot\smile)$ and $(\ddot\frown)$. As illustrated in the diagram, in w_1, which stands for the actual world, Ann is the woman on the left, whereas in w_2, which represents the one possibility in Ralph's doxastic state, Bea is the woman on the left.

There are only two possible conceptual covers definable over such sets of worlds W and individuals D, namely:

$$\begin{array}{lll} \mathrm{A} & = & \{\lambda w[\mathrm{left}]_w, \lambda w[\mathrm{right}]_w\} \\ & & \text{(or equivalently } \{\lambda w[(\ddot\smile)]_w, \lambda w[(\ddot\frown)]_w\}) \\ \mathrm{B} & = & \{\lambda w[\mathrm{Ann}]_w, \lambda w[\mathrm{Bea}]_w\} \end{array}$$

These two covers correspond to the two ways of cross-identifying individuals (i.e. telling of an element of a possible world whether or not it is identical with a given element of another possible world) which are available in such a situation: A cross-identifies those individuals which stand in the same perceptual relation to Ralph. B cross-identifies the women by their name.

All other possible combinations of concepts fail to satisfy the existential or the uniqueness condition. For instance, the set C is not a conceptual cover:

$$\mathrm{C} = \{\lambda w[\mathrm{left}]_w, \lambda w[\mathrm{Ann}]_w\}$$

Formally, C violates both the existential condition (no concept identifies $(\ddot{\frown})$ in w_1) and the uniqueness condition $((\ddot{\smile})$ is counted twice in w_1). Intuitively, the inadequacy of C does not depend on the individual properties of its two elements, but on their combination. Although the two concepts 'the woman on the left' and 'Ann' can both be salient, when contrasted with each other, they cannot be regarded as standing for the two women in the universe of discourse in all relevant worlds.

When we talk about concepts, we implicitly assume two different levels of 'objects': the individuals (in D) and the ways of referring to these individuals (in D^W). An essential feature of the intuitive relation between the two levels of the individuals and of their representations is that to one element of the first set correspond many elements of the second. The intuition behind it is that one individual can be identified in many different ways. What characterizes a set of representations of a certain domain is this cardinality mismatch, which expresses the possibility of considering an individual under different perspectives which may coincide in one world and not in another. Individuals, on the other hand, do not split or merge once we move from one world to the other. Now, since the elements of a cover also cannot merge or split (by uniqueness), they behave like individuals in this sense, rather than representations. On the other hand, a cover is not barely a set of individuals, but encodes information on how these individuals are specified. We thus can think of covers as sets of individuals each identified in one specific way. My proposal is that *de re* belief reports involve quantification over precisely this kind of sets. By allowing different conceptual covers to constitute the domain of quantification on different occasions, we can account for the double vision cases, without failing to account for the intuition that *de re* belief reports involve quantification over genuine individuals, rather than over ways of specifying these individuals.

5.2. *Quantification under Cover*

A *language* $\mathcal{L}_{CC}$ of modal predicate logic under conceptual covers is the language formed out of $\mathcal{L}$ by the addition of a set of new primitive symbols N of conceptual cover indices 0, 1, 2, ... and by changing the definitions of the rules R0 and R4 as follows:

(R0′) (i) If α is a variable in $\mathcal{V}$ and n is a CC-index, α_n is a term in $\mathcal{L}_{CC}$.
(ii) If α is an individual constant in $\mathcal{C}$, α is a term $\mathcal{L}_{CC}$.

(R4′) If ϕ is a wff, x_n is an indexed variable, then $\exists x_n \phi$ is a wff $\mathcal{L}_{CC}$.

CC-indices range over (contextually selected) conceptual covers. I will write $\mathcal{V}_n$ to denote the set of variables indexed with n and $\mathcal{V}_N$ to denote the set $\bigcup_{n \in N}(\mathcal{V}_n)$.

A *model* for $\mathcal{L}_{CC}$ is a quintuple $\langle W, R, D, I, C\rangle$ in which W, R, D, I are as above and C is a set of conceptual covers over (W, D).

DEFINITION 8 (CC-assignment). Let $K = \{f \cup h \mid f \in C^N \,\&\, h \in IC^{\mathcal{V}_N}\}$. A CC-assignment g is an element of K satisfying the following condition: $\forall n \in N$: $\forall x_n \in \mathcal{V}_n$: $g(x_n) \in g(n)$.

In this system, an assignment g has a double role, it works on CC-indices and on indexed variables. CC-indices get assigned conceptual covers in C and n-indexed individual variables, x_n, get assigned concepts in $g(n)$.

The definition of quantification is relativized to conceptual covers. Quantifiers range over elements of contextually determined conceptualizations.

DEFINITION 9 (Quantification under cover).

$$M, w \models_g \exists x_n \phi \quad \text{iff} \quad \exists c \in g(n) : M, w \models_{g[x_n/c]} \phi$$

All other semantic clauses are defined as in CIA, as well as the notion of validity.

Illustration. Consider again the situation described above, with two women standing in front of Ralph. Imagine now that Bea is insane and Ralph is informed about it, Ann is, instead, vaguely known to Ralph as a pillar of the community. He still wrongly thinks that Bea is the woman on the left and Ann is the woman on the right. We can formalize the situation by the following CC model $\langle W, R, D, I', C\rangle$, in which W, R, D are as above, I' is like I with the only addition that Bea is insane in w_1 and in w_2 (in the diagram insanity is represented by a bullet), and C contains the two covers A $= \{\lambda w[\text{left}]_w, \lambda w[\text{right}]_w\}$ and B $= \{\lambda w[\text{Ann}]_w, \lambda w[\text{Bea}]_w\}$ introduced above:

$$w_1 \mapsto \begin{array}{cc} (\ddot{\smile}) & (\ddot{\frown})^\bullet \\ [ann] & [bea] \end{array}$$

$$w_2 \mapsto \begin{array}{cc} (\ddot{\smile})^\bullet & (\ddot{\frown}) \\ [bea] & [ann] \end{array}$$

Consider now the following *de re* sentence:

(56a) Ralph believes Ann to be insane.

(56b) $\exists x_n (x_n = a \wedge \Box I(x_n))$

On the present approach, sentence (56b) obtains different evaluations when interpreted under different conceptual covers. Under an assignment which maps n to cover A, i.e., if the operative conceptual cover is the one which cross-identifies objects by pointing at them, the sentence is true. As a matter of fact, Ann is the woman on the left and Ralph ascribes insanity to the woman on the left.

On the other hand, if the operative cover is the one which cross-identifies objects by their name, than (56b) is false, Ralph indeed believes that Bea is insane, and not Ann.

This variability is in accordance with our intuitions. The acceptability of the sentence is relative to the circumstances of the utterance. For example, (56b) could be correctly uttered as an explanation of Ralph's behaviour, if all of a sudden he starts chasing the woman on the left to bring her to a mental institution. But it might be harder to accept, for example, as an answer to a question about Ralph's general beliefs about Ann.

5.3. *Applications*

The theoretical point behind the present analysis is that natural language *de re* belief reports are about individuals under a perspective. The uniqueness and the existential conditions on conceptual covers exemplify the idea that in *de re* belief reports we never explicitly quantify over ways of specifying objects, but over the objects themselves. On the other hand, the dependency of *de re* belief on the ways of specifying the intended objects is accounted for by allowing different sets of concepts to count as domains of quantification on different occasions. The former feature helps in avoiding the shortest spy problem, the latter provides a solution to the double vision puzzles.

5.3.1. *Double Vision*

Since variables can range over elements of different conceptual covers, the present semantics does not validate the principles **LI** and **LNI** of 'necessary' (non)-identity or their converses. In CC, we can express cases of mistaken identity. The sentences (57) and (58) can be true also in serial models, if the indices n and m are assigned two different covers.

(57) $\exists x_n \exists y_m (x_n \neq y_m \wedge \Box x_n = y_m)$

(58) $\exists x_n \exists y_m (x_n = y_m \wedge \Box x_n \neq y_m)$

The consistency of sentences like (58) shows that we can deal with double vision situations. Recall Quine's Ralph who believes of one man Ortcutt that he is two distinct individuals, because he has seen him in two different circumstances, once with a brown hat, once on the beach. If we want to represent this sort of situation we have to use two different conceptual covers. Representations of cases of mistaken identity crucially involve shifts of conceptualization. This reflects the fact that, intuitively, in order to describe Ralph's misconception, the speaker must assume two different ways of identifying the objects in the domain. According to one way, Ortcutt is identified as the man with the brown hat, according to the other as the man seen on the beach. Shift of covers have a cost, and, therefore, are generally avoided (see Section 5.3.3). The necessity of a plurality of conceptualizations in order to represent double vision cases explains the extraordinary nature of such situations.

In order to see how the specific examples in Quine's story are accounted for in the present framework, consider the simple model $M = \langle W, R, D, C, I \rangle$. W consists of only two worlds, the actual world w_0 and w_1. R is such that w_1 is the only world accessible from w_0. D consists of two individuals Ortcutt o and Porcutt p. In w_0, Ortcutt is the man with the brown hat, but he is also the man seen on the beach. In w_1, Ortcutt is the man on the beach and Porcutt is the man with the brown hat. In both w_0 and w_1, p is a spy and o is not. M can be used to model Quine's situation, by assuming that $\{w_1\} = Bel(w_0)$ represents Ralph's belief state. There are four concepts definable in such a model:

a	**b**	**c**	**d**
$w_0 \mapsto o$	$w_0 \mapsto p$	$w_0 \mapsto o$	$w_0 \mapsto p$
$w_1 \mapsto o$	$w_1 \mapsto p$	$w_1 \mapsto p$	$w_1 \mapsto o$

Concept **a** is the interpretation in M of the description 'the man on the beach'. Concept **c** is the interpretation of the description 'the man with the brown hat'. With respect to W, the concepts **a** and **c** cannot be elements of one and the same cover, because they overlap in w_0 and split in w_1. In order to express Ralph's mistake we have to use the two different covers represented in the following diagrams:

$$\begin{array}{ccc} & & \mathbf{a}\ \ \mathbf{b} \\ \hline w_0 & \mapsto & o\ \ p \\ w_1 & \mapsto & o\ \ p \end{array} \qquad \begin{array}{ccc} & & \mathbf{c}\ \ \mathbf{d} \\ \hline w_0 & \mapsto & o\ \ p \\ w_1 & \mapsto & p\ \ o \end{array}$$

Since we have dropped the assumption that variables within belief contexts refer to bare individuals, we can now give a reasonable answer to Quine's question:

> Can we say of this man (Bernard J. Ortcutt to give him a name) that Ralph believes him to be a spy? (Quine, 1956, p. 179)

Namely, 'It depends'. Question (58) receives a negative or a positive answer relative to the way in which Ortcutt is specified. In the model described above, (59) is true under the assignment that maps x_n to **c** (representing 'the man with the brown hat') and false under the assignment that maps x_n to **a** (which stands for 'the man on the beach'):

(59) $\Box S(x_n)$

As a consequence of this, the following two sentences are true under an assignment which maps n to the cover $\{\mathbf{c}, \mathbf{d}\}$ and m to $\{\mathbf{a}, \mathbf{b}\}$ (we assume that the constant o refers to the individual Ortcutt in w_0):

(60a) Ralph believes Ortcutt to be a spy.

(60b) $\exists x_n(x_n = o \wedge \Box S(x_n))$

(61a) Ralph believes Ortcutt not to be a spy.

(61b) $\exists x_m(x_m = o \wedge \Box\neg S(x_m))$

Sentences (60) and (61) can both be true even in a serial model, but only if n and m are assigned different conceptual covers. This is reasonable, because intuitively one can accept these two sentences without drawing the conclusion that Ralph's beliefs are inconsistent, only if one takes into consideration the two different perspectives under which Ortcutt can be considered. On the other hand, the fact that a shift of cover is required in this case explains the never ending puzzling effect of the Ortcutt story. After reading Quine's description of the facts, both covers (the one identifying Ortcutt as the man with the brown hat, the other identifying Ortcutt as the man on the beach) are equally salient, and this causes bewilderment in the reader who has to choose one of the two in order to interpret each *de re* sentence.

From (60) and (61) we *cannot* conclude the following (for $i \in \{n, m\}$):

(62) $\exists x_i(x_i = o \wedge \Box(S(x_i) \wedge \neg S(x_i)))$

which would charge Ralph with contradictory beliefs. Yet, we can conclude (63) which does not carry such a charge:

(63) $\exists x_n(x_n = o \wedge \exists y_m(o = y_m \wedge \Box(S(x_n) \wedge \neg S(y_m))))$

Consider now what happens to the concepts **a**, i.e. 'the man on the beach' and **c**, 'the man with the brown hat', if restricted to Ralph's belief state $Bel(w_0) = \{w_1\}$:

		a	**c**
w_1	$\mapsto$	o	p

If we restrict our attention to Ralph's doxastic alternatives the two concepts do constitute a conceptual cover, since they exhaust the domain and there is no overlap. But, as we have seen, as soon as we take w_0 into consideration, the set $\{a, c\}$ is no longer a conceptual cover:

		a	**c**
w_0	$\mapsto$	o	o
w_1	$\mapsto$	o	p

The number of definable covers is relative to the number of possible worlds under consideration.[26] A set of concepts that overlap or split with respect to a class of possible worlds, may cease to do this – and so constitute a conceptual cover – with respect to a smaller class of possibilities. Ralph is in a state of maximal (though incorrect) information, in which all covers coincide. In such a state, all possible contrasting methods of cross-identifying the individuals in the universe collapse into one, according to which the man seen on the beach and the man with the brown hat are just two different objects. Indeed, Ralph may reason as follows from his perspective:

(64) The man on the beach is tall. The man with the brown hat is tall. So all relevant people are tall.

From our perspective, however, such a reasoning is flawed. Once we take the actual world w_0 into consideration, we know that the two descriptions are different ways of specifying one and the same object. The concepts **a** and **c** cannot be elements of one and the same cover and hence we cannot quantify over them.

Other double vision puzzles are treated in a similar way, in particular, Kripke's case of Pierre. Recall Pierre is a bilingual who assents to 'Londres est jolie' while denying 'London is pretty', because he is ignorant about the fact that London and Londres are one and the same town. The two relevant sentences are represented as follows:

(65a) Pierre believes that London is pretty.

(65b) $\exists x_n(x_n = l \wedge \Box P(x_n))$

(66a) Pierre believes that London is not pretty.

(66b) $\exists x_m(x_m = l \wedge \neg P(x_m))$

Sentence (65a) can be true only in a *de re* interpretation[27] in which the relevant singular term is interpreted from the speaker's point of view, who knows that London is Londres, rather than from Pierre's perspective who does not know precisely that. Pierre indeed ascribes 'ugliness' to London under the representation 'London', so the *de dicto* reading would be false. But since there is an actual representation of London, namely 'Londres' under which Pierre ascribes it the opposite property, the sentence can be true if interpreted *de re* under the right conceptualization. If we assume that two different covers are operative in the two cases, we account for the intuitive truth of the *de re* sentences (65) and (66) without ascribing Pierre contradictory beliefs.

By means of the same mechanism we can account for the context dependence of *de re* sentences. Since different covers can be assigned to different occurrences of quantifiers, the general principle of renaming is invalidated in the present semantics. The following scheme is *not* valid in CC (where $\phi[x_n/y_m]$ denotes the result of substituting the variable y_m for the variable x_n in the wff ϕ):

PR $\exists x_n\phi \rightarrow \exists x_m\phi[x_n/x_m]$

The relevant counterexamples are sentences in which ϕ contains some belief operator:

(68) $\exists x_n \Box P x_n \not\rightarrow \exists x_m \Box P x_m$

If two different ways of conceptualizing the domain are operative, that is, if two different notions of what counts as a determinate object are assumed, we have no guarantee that if there is a determinate object (according to one cover) such that the subject believes that she is P, then there is a determinate object (according to the other cover) such that the subject believes that she is P.

The failure of **PR** allows us to account for the cases of the theater and Ortcutt discussed in Section 4.2. Let me expand upon the latter. Recall the following examples in which each half of the Ortcutt story is told separately:

(69) Ralph believes that the man with the brown hat is a spy.
The man with the brown hat is Ortcutt.
So Ralph believes of Ortcutt that he is a spy.

(70) Ralph believes that the man seen on the beach is not a spy.
The man seen on the beach is Ortcutt.
So Ralph does *not* believe of Ortcutt that he is a spy.

Consider now the two relevant *de re* sentences:

(71) $\exists x_n(x_n = o \wedge \Box S(x_n))$

(72) $\neg\exists x_m(x_m = o \wedge \Box S(x_m))$

The two sentences do not contradict each other, if n and m are assigned two different covers. And since covers are pragmatically chosen, two different covers can be selected in the two circumstances. The story in (69) in which the concept 'the man with the brown hat' has been explicitly introduced strongly suggests a cover containing this representation (e.g., cover $\{c, d\}$ in our model). Whereas a cover containing 'the man seen on the beach' (e.g., cover $\{a, b\}$ in our model) is made salient by the previous discourse in the second case (70). Finally note that (72) and (73) turn out to be equivalent in the present framework, like in classical MPL.

(73) $\exists x_m(x_m = o \wedge \neg\Box S(x_m))$

This distinguishes the present semantics from the two CI semantics discussed above, which, as we saw, predicted a structural ambiguity for sentences like 'Ralph does *not* believe of Ortcutt that he is a spy'. Note that it was the presence of overlapping concepts, which express the possibility of considering an object simultaneously under different perspectives, which gave rise to the dubious ambiguity in the CI systems. If each object is identified by one and only one concept, the two readings collapse, as we intuitively expect. On the present account, negations of *de re* belief reports are not structurally ambiguous, but, like their positive counterparts, they are simply context dependent.

5.3.2. *The Shortest Spy*

The present semantics obviously does not validate **EX** or **TEX**. Existential generalization (and term exportation) can be applied to a term t occurring in the scope of a belief operator only with the extra premise that there is a concept c in the operative cover such that t denotes instantiations of c in all doxastic alternatives of the relevant agent (plus the actual world). This is precisely what the following two CC-valid principles say:[28]

$$\textbf{EG}_{\Box n}\quad \exists x_n \Box t = x_n \rightarrow (\Box\phi[t] \rightarrow \exists x_n \Box\phi[x_n])$$

$$\textbf{TEX}_{\Box n}\quad \exists x_n(x_n = t \wedge \Box x_n = t) \rightarrow (\Box\phi[t] \rightarrow \exists x_n(x_n = t \wedge \Box\phi[x_n]))$$

As in MPL, a term occurring in a belief context must denote one and the same determinate object in all of the relevant worlds in order for existential generalization or term exportation to be applicable to it. But unlike in MPL, on the present account, the notion of a determinate object is not left unanalyzed. What counts as an object is not given *a priori*, but depends on the operative cover, which is contextually determined. The parallelism between standard modal predicate logic and the present semantics with respect to (term) exportation is sufficient to solve the shortest spy problem at least to a certain extent.

(74a) Ralph believes that there are spies.

(74b) $\Box\exists x_n S(x_n)$

(75a) There is someone whom Ralph believes to be a spy.

(75b) $\exists x_n \Box S(x_n)$

(76a) Ralph believes that the president of Russia is the president of Russia.

(76b) $\Box r = r$

(77a) Ralph believes Putin to be the president of Russia.

(77b) $\exists x_m(x_m = p \wedge \Box x_m = r)$

As in MPL, (74) and (76) express *de dicto* readings with possibly different determinate objects – according to the operative cover – being spies or presidents in different worlds in Ralph's belief state; whereas (75) and (77) express *de re* readings, in which one and the same determinate object – according to the operative cover – is ascribed the relevant property in all relevant worlds. Example (75) does not follow from (74), and (77) does not follow from (76) (plus the assumption $p = r$), because the relevant conceptual covers do not have to include problematic concepts like λw[the shortest spy]$_w$ or λw[the president of Russia]$_w$ respectively. Still, our semantics allows statements like (75) and (77) to be true in circumstances in which they are obviously deviant, like in the situations where no-one in particular is believed by Ralph to be a spy or to be the president of Russia (see Section 4.1, examples (33) and (35)). Even in such situations, we have no trouble in finding values for n and for m under which (75) and (77) are true, namely any two covers containing the concept λw[the shortest spy]$_w$ and λw[the president of Russia]$_w$ respectively. It is not immediately obvious how we can rule out such problematic assignments. Ruling them out by not including the problematic covers in the set C in our model, is not a viable option, because of the problems illustrated by the examples of Odette's lover or of Susan's mother discussed in Section 4.2. Recall the relevant sentences in the latter case:

(78) He must think I am rich.

(79) He must think I am your mother.

If, in order to explain the inadequacy of (79) in the described situation, we rule out any cover containing the concept λw[Susan's mother]$_w$, then we are unable to account for the truth of (78) in which such a concept is crucially quantified over. Hintikka (1962) (or MPL) and Kaplan (1969) (or CIB) cannot account for these cases. In the present pragmatic account, on the other hand, they receive a natural explanation. In CC, domains are contextually selected. As we will see in the following section, the selection of a domain containing λw[Susan's mother]$_w$ is optimal for sentence (79), but not for sentence (78), unless one is willing to violate important pragmatic constraints. Let us have a closer look.

5.3.3. *Pragmatic Aspects of Conceptual Covers*

In this article, I have argued that a number of seeming paradoxes emerging from logical analyses of propositional attitudes can be explained in terms of shifts in perspectives over the universe of discourse. In this section, I would like to briefly address the issue of how different perspectives are selected on different occasions.

Shifts in perspective have a cost. They require reasonings and adjusting and, therefore, are generally avoided. However, on certain occasions, like in Quine's double vision puzzle or in the case of Susan's mother, we do shift the domain of quantification. In this section I would like to argue that in these cases we are compelled to make such shifts by the requirement to comply with general pragmatic principles of interpretation and generation.[29]

As a first illustration consider Quine's double vision example, where Grice's *Maxim of Quality* plays a crucial role.

(80a) Ralph believes Ortcutt to be a spy and Ralph believes Ortcutt not to be a spy.

(80b) $\exists x_n(x_n = o \wedge \Box S(x_n)) \wedge \exists y_m(y_m = o \wedge \Box\neg S(y_m))$

In this example, a general economy principle would suggest to interpret m as n. However, the fulfillment of Quality prevents this resolution, because it would render the sentence inconsistent with the common ground. If m is n, the sentence would say that Ralph's beliefs are contradictory, but from Quine's story it is known that this is not the case.

Quality. Try to make your contribution one that is true.

Let us now turn to the example of the shortest spy. Consider sentence (81) used in the situation described by Kaplan (see example (33) above).

(81a) There is someone whom Ralph believes to be a spy.

(81b) $\exists x_n \Box S(x_n)$

Kaplan's description of the context suggests as operative a conceptual cover which does *not* contain the non-rigid concept λw[the shortest spy]$_w$. While interpreting this sentence we have then two relevant options. Either (i) we accommodate a new domain which contains λw[the shortest spy]$_w$ or (ii) we do not. Option (ii) leads to an interpretation which is inconsistent with the background information, so it leads to a violation of Grice's Quality Maxim. Still we do not accommodate. Why? Because accommodating here leads to more severe principle violations. First of all option (i) leads to a violation of another maxim of rational conversation, namely Grice's *Maxim of Quantity* – the sentence would be trivialized –, and, secondly, it fails to fulfil our economy principle which prohibits shifts of quantificational domains.

Quantity. Make your contribution as informative as is required.

The same kind of pragmatic explanation can be employed in order to account for the unacceptability of (82) if used in the situation described by van Fraassen (see examples (41) and (42) above).

(82) He must think I am your mother.

Also in this case we choose not to accommodate a new domain even when this leads to an inconsistency with the common ground. This is because the only accommodation which would save consistency, namely the accommodation of a domain containing the non-rigid concept λw[Susan's mother]$_w$, would trivialize the sentence. Accommodating is therefore ruled out by Quantity. Consider now van Fraassen's original example.

(83) He must think I am rich.

The intended interpretation for this sentence could be paraphrased as 'I am Susan's mother and he would assent to "Susan's mother is rich".' To obtain this interpretation we must accommodate a non-rigid domain containing λw[Susan's mother]$_w$. Note that such accommodation here does not trivialize the sentence, and therefore, in contrast to (82), is not ruled out by Quantity. Can we then conclude that we can always accommodate a non-standard domain in order to save consistency when this does not lead to a trivial content? No. Compare example (83) with the following variation of Kaplan's example of the shortest spy.

Assume as part of the common ground the information that Ortcutt is the shortest spy, that Ralph does not believe of Ortcutt that he is a spy, and that Ralph would not assent to 'Ortcutt is fat'. Consider the following sentence:

(84) Ralph believes that Ortcutt is fat.

As in example (83), also here only the accommodation of a non-rigid domain, namely a domain containing λw[the shortest spy]$_w$, would lead to a consistent interpretation, namely the interpretation which can be paraphrased as '(Ortcutt is the shortest spy and) Ralph would assent to "The shortest spy is fat".' However, people do not accept to accommodate in this case. Sentence (84) is deviant in the given context and should be rejected. How is this example different from the case of Susan's mother? In both cases only the accommodation of a non-rigid domain leads to a consistent interpretation. But only in the case of Susan's mother we do accept to accommodate. Why?

We can interpret sentence (83) as saying 'He would assent to "Susan's mother is rich"', while we are not ready to interpret sentence (84) as 'Ralph would assent to "The shortest spy is fat"'. The explanation I would like to propose for why this last interpretation is not assigned to (84) in the described situation, is that a speaker, who had used such a sentence to express such a content, would not have been cooperative. Indeed, the same content could have been conveyed in a much more efficient way by means of an alternative form, namely (85).[30]

(85) Ralph believes that *the shortest spy* is fat.

The existence of an alternative more efficient expression for the content under discussion seems to prevent its selection as an acceptable interpretation for (84).[31]

The case of Susan's mother is crucially different. Although also here the chosen formulation in (83) requires the accommodation of a new domain of quantification, it is hard to find a more efficient form for the intended content. In that situation Susan's mother could have used sentence (86) instead of (83) to say what she wanted to say and she would have been more cooperative. But the former sentence would not have been as good as the latter given the circumstance for utterance.

(86) He must think *Susan's/your mother* is rich.

There are a number of reasons why (86) is not as good as (83) in this context. In particular, (86) seems to crucially violate a principle which requires the use of the pronoun 'I' to refer to the speaker in a conversation. We can derive this principle from a more general one, which I will call the *Referential Device Principle (RDP)*. This assumes the hierarchy of referential devices in (87) from (Zeevat, 2002) (with the addition of the clause for proper names which is mine).

RDP. A referential device can be selected only if the application criteria of the classes above in the following hierarchy do not apply.

(87)

NP type	Selection condition
reflexive	c-command
1st and 2nd pers. pron.	conversation participant
demonstratives anaphoric short definites proper names ...	presence in attention space high salience through mention old, dependence on high salient familiarity ...
long definites	new and unique
indefinites	new

RDP is typically a generation constraint, but it can also influence interpretation, as is shown by the following example from Grice (also discussed in (Zeevat, 2002)).

(88) X is meeting a woman this evening.

According to Grice, sentence (88) has the implicature that the woman under discussion cannot be known to be X's mother, or sister, and, if we follow the hierarchy in (87), we can further infer that she is not the speaker, the addressee, ...

The following case illustrates how Zeevat's hierarchy seems to influence our ways of reporting (and interpreting) propositional attitudes.

Lorenzo's Mother. Consider the following situation. Miss Jones, the new director of Lorenzo's school, would assent to 'Lorenzo's mother is Spanish'. However, she has no idea who Lorenzo's mother is. Lorenzo's mother's name is Maria. Consider now the following sentences used to report the described situation in the following contexts.

(C1) Maria to her husband:

(89) The new director of Lorenzo's school believes that *I* am Spanish.

(C2) Maria's husband to Maria:

(90) The new director of Lorenzo's school believes that *you* are Spanish.

(C3) Maria's husband to her mother:

(91) The new director of Lorenzo's school believes that *Maria* is Spanish.

(C4) Lorenzo's teacher to a colleague:

(92a) Miss Jones believes that *she* [pointing at Maria] is Spanish.

(92b) (Lorenzo's mother is Italian, but) Miss Jones believes that *she* is Spanish.

(92c) (?) Miss Jones believes that *Maria* is Spanish.

All these examples, with the only exception of (92c), seem to be adequate ways for describing the situation in the given circumstances, even if, under the intended interpretation, they all violate our economy principle – a domain containing the non-rigid concept λw[Lorenzo's mother]$_w$ must be accommodated. The following alternative formulation for the same content would have avoided such a violation.[32]

(93) Miss Jones/The new director believes that *Lorenzo's mother* is Spanish.

On the other hand, sentence (93) does not seem to be a live option for the speaker in any but one of the considered contexts. Example (93) indeed violates principle RDP in all contexts but C4(c), where the familiarity condition for the use of a proper name is not satisfied. In contexts C1–C4(b) then, sentences (89)–(92b) are among the best candidates the speaker could have chosen in order to express what (s)he wanted to express on that occasion. In context C4(c), example (92c) is not among the best choices. Therefore, I would like to suggest, we accept the accommodation of a non-standard domain of quantification only in the former cases. Example (92c) in C4(c) is then unacceptable because the only interpretation which would make it true, namely 'Miss Jones would assent to "Lorenzo's mother is Spanish" ' is ruled out by the alternative candidate (93) which, as we have said, in C4(c) can express that content without any constraint violation.

Example (89) has exactly the same structure as the example of Susan's mother, while example (92c), I would like to suggest, is parallel to the example of the fat shortest spy.

(94) He must think I am rich.

(95) Ralph believes that Ortcutt is fat.

Example (94) is acceptable in the situation described by van Fraassen because the alternative form containing the long description 'Susan's mother', although more cooperative, since it would not involve accommodation, is not a live option for the speaker. Example (95) is not acceptable in the described context, because there is an alternative form for the intended content which is strictly better, namely the form containing the description 'the shortest spy', which does not require any domain accommodation, and does not violate the Referential Device Principle. There is no reason to prefer 'Ortcutt' over 'the shortest spy' as a referential device in this case, because no familiarity is assumed between us (readers of a philosophical article) and Ortcutt. It should be clear that on the present account, the difference between these two examples does not rely on the nature of the cognitive relation between the subject and the object of belief or on the appropriateness of the concept λw[Susan's mother]$_w$ *versus* λw[the shortest spy]$_w$, but crucially on the circumstances of the utterance. If we slightly change these circumstances, for example, by assuming in the case of the fat shortest spy that the participants to the conversation are familiar with Ortcutt or that Ortcutt himself is speaking, we predict, I believe correctly, that concept λw[the shortest spy]$_w$ could be part of our domain of quantification. The former case would be similar to example (91) and the latter to example (89) above.

I would like to conclude this section with a brief comment on the relation between *de re* attitudes and knowing-who constructions. On Hintikka's (1962) and Kaplan's (1969) accounts, having a *de re* attitude requires knowing who somebody is. This seems correct in many cases, like the case of the fat shortest spy, but not all. In examples like those of Susan's mother or Odette's lover,[33] having a *de re* attitude does not require knowing who somebody is under any correct interpretation of the latter notion.[34] On the present analysis, these are cases in which a non-rigid domain of quantification is accommodated. Such accommodation, I have argued, must occur for a reason, namely to avoid violations of general principles of conversation. 'Ordinary' cases, like the case of the fat shortest spy, in which having a *de re* attitude does require knowing who somebody is, are cases where no such reason is available, and, therefore, no shift to a non-rigid domain is justified.

5.4. *MPL and CC Validity*

In this section, I compare CC with ordinary MPL. I will show that, if there are no shifts of conceptual covers, modal predicate logic under conceptual covers is just ordinary modal predicate logic, the two types of semantics turn out to define exactly the same notion of validity. Once we allow shifts of covers though, a number of problematic MPL-valid principles cease to hold.

I call a model for $\mathcal{L}_{CC}$ containing a single conceptual cover a classical model:

DEFINITION 10 (Classical CC-models). Let $M = \langle W, R, D, I, C\rangle$ be a model for a language $\mathcal{L}_{CC}$ of modal predicate logic under conceptual covers.

$$M \text{ is } \textit{classical} \text{ iff } |C| = 1$$

I define a notion of classical CC-validity. A formula is classically valid iff it is valid in all classical CC-models.

DEFINITION 11 (Classical CC-validity). Let ϕ be a wff in $\mathcal{L}_{CC}$.

$$\models_{CCC} \phi \quad \text{iff} \quad \forall M : M \text{ is classical } \Rightarrow M \models_{CC} \phi$$

The main result of this section is that, if we just consider classical models, the logic of conceptual covers does not add anything to ordinary modal predicate logic. Classical CC-validity is just ordinary MPL-validity. This result is expressed by the following proposition where ϕ is a wff in $\mathcal{L}_{CC}$ which is clearly also interpretable in modal predicate logic:[35]

PROPOSITION 2. *Let ϕ be a wff in $\mathcal{L}_{CC}$.*

$$\models_{CCC} \phi \quad \textit{iff} \quad \models_{MPL} \phi$$

One direction of the proof of this proposition follows from the fact that given a classical CC-model M, we can define an equivalent ordinary MPL-model M', that is, an MPL-model that satisfies the same wffs as M. Let M be $\langle W, R, D, I, \{CC\}\rangle$. We define an equivalent model $M' = \langle W', R', D', I'\rangle$ as follows. $W' = W$, $R' = R$, $D' = CC$. For I' we proceed as follows.[36]

(i) $\forall\langle c_1, \ldots, c_n\rangle \in CC^n, w \in W, P \in \mathcal{P}$:

$$\langle c_1, \ldots, c_n\rangle \in I'(P)(w) \quad \text{iff} \quad \langle c_1(w), \ldots, c_n(w)\rangle \in I(P)(w)$$

(ii) $\forall c \in CC, w \in W, a \in \mathcal{C}$:

$$I'(a)(w) = c \quad \text{iff} \quad I(a)(w) = c(w)$$

In our construction, we take the elements of the conceptual cover in the old model to be the individuals in the new model, and we stipulate that they do, in all w, what their instantiations in w do in the old model. Clause (i) says that a sequence of individuals is in the denotation of a relation P in w in the new model iff the sequence of their instantiations in w is in P in w in the old model. In order for clause (ii) to be well-defined, it is essential that CC is a conceptual cover, rather than an arbitrary set of concepts. In M', an individual constant a will denote in w the unique c in CC such that $I(a)(w) = c(w)$. That there is such a unique c is guaranteed by the uniqueness condition on conceptual covers. We have to prove that this construction works. I will use g, g' for assignments within M and h, h' for assignments within M'. Note that for all assignments g within M: $g(n) = CC$ for all CC-indices n, since CC is the unique cover available in M. I will say that g corresponds with h iff $g = h \cup \{\langle n, CC\rangle \mid n \in N\}$. This means that the two assignments assign the same value to all individual variables x_n for all n, and g assigns the cover CC to all CC-indices n. In the Appendix we prove the following theorem.

THEOREM 1. *Let g and h be any corresponding assignments. Let w be any world in W and ϕ any wff in $\mathcal{L}_{CC}$. Then*

$$M, w, g \models_{CC} \phi \quad \textit{iff} \quad M', w, h \models_{MPL} \phi$$

Now it is clear that if a classical CC-model M and an ordinary MPL-model M' correspond in the way described, then the theorem entails that any wff in $\mathcal{L}_{CC}$ is CC-valid in M iff it is MPL-valid in M'. Thus, given a classical CC-model, we can define an equivalent MPL-model, but also given an MPL-model, we can define an equivalent classical CC-model $\langle W, R, D, \{CC\}, I\rangle$ by taking CC to be the rigid cover. This suffices to prove Proposition 2.

A corollary of Proposition 2 is that CC-validity is weaker than MPL-validity. $\models_{CC} \phi$ obviously implies $\models_{CCC} \phi$ which by Proposition 2 implies $\models_{MPL} \phi$.

COROLLARY 1. *If $\models_{CC} \phi$, then $\models_{MPL} \phi$.*

A further consequence of Proposition 2 is that we can define interesting fragments of $\mathcal{L}_{CC}$ which behave classically, that is, wffs of these fragments are valid iff they are valid in MPL. This is done in the following two propositions.

PROPOSITION 3. *Let $\mathcal{L}^n_{CC}$ be a restriction of $\mathcal{L}_{CC}$ containing only variables indexed by n, and $\phi \in \mathcal{L}^n_{CC}$. Then $\models_{CC} \phi$ iff $\models_{MPL} \phi$.*

Proof. Suppose $\not\models_{CC} \phi$ for $\phi \in \mathcal{L}^n_{CC}$. This means for some CC-model $M = \langle W, R, D, C, I\rangle$ and some w,g: $M, w \not\models_g \phi$. Let $M' = \langle W, R, D, \{g(n)\}, I\rangle$. Since ϕ can only contain variables indexed by n, $M', w \not\models_g \phi$. M' is obviously a classical model. This means $\not\models_{CCC} \phi$ which by Proposition 2 implies $\not\models_{MPL} \phi$. Corollary 1 delivers the second half of Proposition 3. □

PROPOSITION 4. *Let $\mathcal{L}_{PL}$ be the non-modal fragment of $\mathcal{L}_{CC}$, and $\phi \in \mathcal{L}_{PL}$. Then $\models_{CC} \phi$ iff $\models_{MPL} \phi$.*

Proof. Suppose $\not\models_{CC} \phi$. This means for some CC-model $M = \langle W, R, D, C, I\rangle$ and some w,g: $M, w \not\models_g \phi$. Let $M' = \langle W', R', D, C', I'\rangle$, be a sub-model of M such that $W' = \{w\}$. Since ϕ is non-modal $M', w \not\models_g \phi$. Since $|W'| = 1$, $|C'| = 1$, i.e. M' is a classical model. This means $\not\models_{CCC} \phi$ which by Proposition 2 implies $\not\models_{MPL} \phi$. Again Corollary 1 delivers the other direction of the proof. □

As a consequence of Proposition 4, our CC semantics validates the principles of existential generalization and substitutivity of identicals for non-modal wffs, since they are validated in MPL:

SI1 $\models_{CC} t = t' \rightarrow (\phi[t] \rightarrow \phi[t'])$ (if ϕ is non-modal)

EG1 $\models_{CC} \phi[t] \rightarrow \exists x_n \phi[x_n]$ (if ϕ is non-modal)

Note that the validity of **EG1** crucially relies on the existence condition on conceptual covers, which guarantees that whatever denotation, $d = [t]_{M,g,w}$, a term t is assigned to in w, there is a concept c in the operative cover such that $c(w) = d = [t]_{M,g,w}$.

Substitutivity of identicals and existential generalization cease to hold as soon as we introduce belief operators. By Corollary 1, **SI** and **EG** are invalidated in CC, being invalid in MPL:

SI $\not\models_{CC} t = t' \rightarrow (\phi[t] \rightarrow \phi[t'])$

EG $\not\models_{CC} \phi[t] \rightarrow \exists x_n \phi[x_n]$

The failures of **SI** and **EG** are welcome because they allow us to solve the *de dicto* substitutivity puzzles (see example (5) in Section 2) and the shortest spy problems:

(96) $t = t' \not\Rightarrow (\Box Pt \rightarrow \Box Pt')$

(97) $\Box Pt \not\Rightarrow \exists x_n \Box P x_n$

It is easy to show that not only **SI** and **EG** can fail, but also **SIv** and **EGv** are invalidated in the present semantics.

SIv $\not\models_{CC} x_n = y_m \rightarrow (\phi[x_n] \rightarrow \phi[y_m])$

EGv $\not\models_{CC} \phi[y_m] \rightarrow \exists x_n \phi[x_n]$

From the failure of **SIv**, it follows that also **LIv** is not valid in CC:

(98) $x_n = y_m \not\Rightarrow \Box\, x_n = y_m$

And this allows us to model cases of mistaken identity and to solve the double vision problems.

From the failure of **EGv**, it follows that also the principle of renaming **PR** is not generally valid in CC:

(99) $\exists x_n \Box P(x_n) \not\Leftrightarrow \exists y_m \Box P(y_m)$

And this allows us to deal with the case of Odette's lover and other cases of context sensitivity.

Note finally that substitutivity of identicals and existential generalization are allowed when applied to variables with a uniform index. It is easy to see that the present semantics validates the following schemes:

SIn $\models_{CC} x_n = y_n \rightarrow (\phi[x_n] \rightarrow \phi[y_n])$

EGn $\models_{CC} \phi[y_n] \rightarrow \exists x_n \phi[x_n]$

The validity of **SIn** crucially relies on the uniqueness condition on conceptual covers. From **SIn**, but also as a consequence of Proposition 3, we can derive **LIn**, which guarantees that the elements in our domains of quantification behave more like individuals than representations:

LIn $\models_{CC} x_n = y_n \rightarrow \Box x_n = y_n$

In this section we have seen that modal predicate logic under conceptual covers is essentially richer than standard MPL because in the former we can shift from one cover to another. If we stick to one cover, not only CC and MPL define the same notion of validity (Proposition 2), but also, and maybe more significantly, the same notion of truth (Theorem 1). We have already seen the intuitive consequences of this result. On the one hand, in ordinary situations in which the method of identification is kept constant, CC behaves exactly as MPL and inherits its desirable properties (for example, in relation to the shortest spy problem). On the other hand, the system is flexible enough to account for extraordinary situations as well, such as double vision situations as well as those like Odette's lover or Susan's mother which are situations in which multiple covers are operative.

So far we have studied the issue of belief attribution from a model theoretic perspective. In the next section we turn to the proof theoretic perspective from which modal logic originated.

5.5. *Axiomatization*

In this section, I present an axiom system which, as proved in the Appendix, provides a sound and complete characterization of the set of wffs valid in all CC-models.

The system CC consists of the following set of axiom schemata:[37]

Basic Propositional Modal System

PC All propositional tautologies.
K $\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$

Quantifiers. Recall that $\phi[t]$ and $\phi[t']$ differ only in that the former contains the term t in one or more places where the latter contains t'.

EGa $\phi[t] \to \exists x_n \phi[x_n]$ (if ϕ is atomic)
EGn $\phi[y_n] \to \exists x_n \phi[x_n]$
BFn $\forall x_n \Box\phi \to \Box\forall x_n \phi$

Identity

ID $t = t$
SIa $t = t' \to (\phi[t] \to \phi[t'])$ (if ϕ is atomic)
SIn $x_n = y_n \to (\phi[x_n] \to \phi[y_n])$
LNIn $x_n \neq y_n \to \Box\, x_n \neq y_n$

Let AX_{CC} be the set of axioms of CC. The set of CC-theorems T_{CC} is the smallest set such that:

AX $AX_{CC} \subseteq T_{CC}$
MP If ϕ and $\phi \to \psi \in T_{CC}$, then $\psi \in T_{CC}$
∃I If $\phi \to \psi \in T_{CC}$ and x^n not free in ψ, then $(\exists x_n \phi) \to \psi \in T_{CC}$
N If $\phi \in T_{CC}$, then $\Box\phi \in T_{CC}$

I will use the standard notation and write $\vdash_{CC} \phi$ for $\phi \in T_{CC}$.

The axioms **EGa** and **SIa** govern existential generalization and substitutivity of identicals for arbitrary singular terms in atomic formulae. **EGn** and **SIn** cover the case for simple variables for general formulae. Note that **EGa** expresses the existence condition on conceptual cover and **SIn** the uniqueness condition.

In atomic contexts, existential generalization is applicable to any term (**EGa**), and any two co-referential terms are interchangeable *salva veritate* (**SIa**). This can be generalized to any non-modal context. In the CC system, we can deduce **EG1** and **SI1**:[38]

EG1 $\vdash_{CC} \phi[t] \rightarrow \exists x_n \phi[x_n]$ (if ϕ is non-modal)

SI1 $\vdash_{CC} t_1 = t_2 \rightarrow (\phi[t_1] \rightarrow \phi[t_2])$ (if ϕ is non-modal)

On the other hand, any n-indexed variable occurring in any arbitrary context is suitable for n-existential generalization (**EGn**), and any two co-referring variables indexed in a uniform way can be substituted *salva veritate* in any context (**SIn**).

There is another pair of related theorems derivable in CC, which govern existential generalization and substitutivity of identicals for formulae with one layer of modal operators:[39]

EG$_{\Box n}$ $\vdash_{CC} \exists x_n \Box t = x_n \rightarrow (\Box\phi[t] \rightarrow \exists x_n \Box\phi[x_n])$ (if ϕ is non-modal)

SI$_{\Box}$ $\vdash_{CC} \Box t_1 = t_2 \rightarrow (\Box\phi[t_1] \rightarrow \Box\phi[t_2])$ (if ϕ is non-modal)

If we add to our axiomatic base the principles **D**, **4** and **E**, these two theorems can be generalized to any ϕ as we expect to be the case for a logic of belief.

Finally note that **BFn** and **LNIn** have the property that they are derivable for some other choices of the basic propositional modal system, e.g., B or S5. I will not consider those systems though, because **B** is not a plausible principle for a logic of belief, so we have to take **BFn** and **LNIn** as axioms. **LIn** is instead derivable in CC, as well as the n-versions of the converse of the Barcan formula and the principle of importation. The proofs are standard.

LIn $\vdash_{CC} x_n = y_n \rightarrow \Box\, x_n = y_n$

CBFn $\vdash_{CC} \Box\forall x_n \phi \rightarrow \forall x_n \Box\phi$

IMn $\vdash_{CC} \exists x_n \Box\phi \rightarrow \Box\exists x_n \phi$

The converse of **IMn**, instead, is *not* provable and it is not valid indeed.

EXn $\nvdash_{CC} \exists x_n \Box\phi \rightarrow \Box\exists x_n \phi$

In the Appendix, we prove that the system CC is sound and complete with respect to the set of all CC-models. By standard techniques we can show that CC $+$ **D** $+$ **4** $+$ **E** is sound and complete with respect to all serial, transitive and Euclidean CC-models.

6. Synopsis

The following diagram summarizes the content of this article. On the topmost horizontal row, the four systems are displayed that we have encountered in the previous sections. On the second column from the left, the principles we have discussed are listed. On the leftmost column, the problems are reported, which are caused by the validity of these principles. The $\models_{(*)}$ or $\not\models_{(*)}$ indicate that the relevant systems do or do not validate the corresponding principles and that this is problematic.

		MPL	CIA	CIB	CC
de dicto substitutivity puzzles	**SI**	$\not\models$	$\not\models$	$\not\models$	$\not\models$
	SI1	$\models$	$\models$	$\models$	$\models$
(*) double vision problems **LIv**	**SIv**	$\models_{(*)}$	$\not\models$	$\not\models$	$\not\models$
	SIn				$\models$
(*) shortest spy problems **(T)EX**	**EG**	$\not\models$	$\models_{(*)}$	$\not\models$	$\not\models$
	EG1	$\models$	$\models$	(?) $\not\models_{(*)}$	$\models$
(*) Odette's lover and other problems **PR**	**EGv**	$\models_{(*)}$	$\models_{(*)}$	$\models_{(*)}$	$\not\models$
	EGn				$\models$

We started by discussing the principles of substitutivity of identical **SI** and existential generalization **EG**. **SI** fails in all considered systems, once they interpret individual constants as non-rigid designators. The failure of **SI** allows us to avoid the *de dicto* substitutivity puzzles. The difficulty of MPL was that it failed to account for the dependence of belief on the ways of specifying objects and, therefore, it ran in the double vision problems (by verifying **SIv**, MPL verifies **LIv**). The CIA solution to these problems consisted in letting variables range over all individual concepts rather than all objects (**SIv** is falsified in CIA). However such a strategy led directly to the shortest spy problems (principles **EG** and **(T)EX** are validated in CIA). CIB solved both problems by letting variables range over suitable subsets of the set of all individual concepts (**SIv** and **EG** are not CIB-valid). But, since the information about the suitable concepts was determined by the model, rather than by a contextual parameter, the system could not avoid the problem of Odette's lover and in general could not account for the context sensitivity of *de re* constructions (**EGv** and so **PR** are validated in CIB). Furthermore, without further refinement, CIB does invalidate **EG1**, which is highly counter-intuitive (see discussion around **EG1** in Section 4.3).

The CC analysis solves these problems by staying as close as possible to MPL. In CIA and CIB, variables range over sets not governed by the principle of substitutivity of identicals (**EGv** holds, whereas **SIv** fails), so typically over sets of representations. On the other hand, in MPL and CC, the 'objects' over which we quantify, cannot split once we move from one world to the other (**EGv** and **SIv** hold in MPL, and **EGn** and **SIn** hold in CC), and therefore behave like individuals, rather than representations of individuals. But while in MPL, the validity of **SIv** and **EGv** led to the double vision and the problem of Odette's lover respectively, in CC, only the weaker **SIn** and **EGn** are validated. **SIv** and **EGv** can fail and, therefore, cases of mistaken identity and of context sensitivity can be accounted for.

7. Conclusion

Many authors have recognized the availability of different methods of cross-identification, and argued that in different contexts different methods can be used. The present analysis was an attempt to give a precise formalization of this insight and to discuss its impact on the interpretation of *de re* belief attributions. By taking variables to range over elements of contextually selected conceptual covers, we account for the ordinary sense of belief, according to which belief attributions depend on ways of specifying objects, while avoiding the counterintuitive results which arise when we quantify over ways of specifying individuals rather than over the individuals themselves.

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APPENDIX: PROOFS

THEOREM 1. *1 Let g and h be any corresponding assignments. Let w be any world in W and ϕ any wff in $\mathcal{L}_{CC}$. Then*

$$M, w, g \models_{CC} \phi \quad \textit{iff} \quad M', w, h \models_{MPL} \phi$$

Proof. The proof is by induction on the construction of ϕ. We start by showing that the following holds for all terms t:

(A) $[t]_{M,w,g} = [t]_{M',w,h}(w)$

Suppose t is a variable. Then $[t]_{M,w,g} = g(t)(w)$. By definition of corresponding assignments, $g(t)(w) = h(t)(w)$. Since $[t]_{M',w,h} = h(t)$, this means that $[t]_{M,w,g} = [t]_{M',w,h}(w)$. Suppose now t is a constant. Then $[t]_{M,w,g} = I(t)(w)$. By the existence and uniqueness conditions on conceptual covers, there is a unique $c \in CC$, such that $I(t)(w) = c(w)$. By clause (ii) of the definition of I': $I'(t)(w) = c$. Since $[t]_{M',w,h} = I'(t)(w)$, this means $[t]_{M,w,g} = c(w) = [t]_{M',w,h}(w)$. We can now prove the theorem for atomic formulae.

Suppose ϕ is $Pt_1, \ldots, t_n$. Now $M, w, g \models_{CC} Pt_1, \ldots, t_n$ holds iff (a) holds:

(a) $\langle [t_1]_{M,w,g}, \ldots, [t_n]_{M,w,g} \rangle \in I(P)(w)$.

By (A), (a) holds iff (b) holds:

(b) $\langle [t_1]_{M',w,h}(w), \ldots, [t_n]_{M',w,h}(w) \rangle \in I(P)(w)$

which, by definition of I', is the case iff (c) holds:

(c) $\langle [t_1]_{M',w,h}, \ldots, [t_n]_{M',w,h} \rangle \in I'(P)(w)$

which means that $M', w, h \models_{MPL} Pt_1, \ldots, t_n$.

Suppose now ϕ is $t_1 = t_2$. $M, w, g \models_{CC} t_1 = t_2$ holds iff (d) holds:

(d) $[t_1]_{M,w,g} = [t_2]_{M,w,g}$

By (A) above, (d) holds iff (e) holds:

(e) $[t_1]_{M',w,h}(w) = [t_2]_{M',w,h}(w)$

which, by the uniqueness condition on conceptual covers, is the case iff (f) holds:

(f) $[t_1]_{M',w,h} = [t_2]_{M',w,h}$

which means that $M', w, h \models_{MPL} t_1 = t_2$.

The induction for $\neg$, $\exists$, $\wedge$ and $\Box$ is immediate. □

Soundness

THEOREM 2 (Soundness). *If* $\vdash_{CC} \phi$*, then* $\models_{CC} \phi$.

Proof. The proof that **MP**, ∃**I** and **N** preserve validity is standard. The validity of **PC** and **K** is obvious. The validity of **EGa** and **SIa** follows from Proposition 4. **LNIn** is valid by Proposition 3. The validity of **SIn** may be established by induction on the construction of ϕ. We show that **EGn** and **BFn** are valid.

EGn Suppose $M, w \models_g \phi[y_n]$. This implies $M, w \models_{g[x_n/g(y_n)]} \phi[x_n]$. Since $g(y_n) \in g(n)$, $M, w \models_g \exists x_n \phi[x_n]$.

BFn Suppose $M, w \models_g \forall x_n \Box\phi$. Let h be any x_n-alternative of g, $g[x_n]h$, i.e. let h be such that: $\forall v \in (N \cup \mathcal{V}_N)$: $v \neq x_n \Rightarrow g(v) = h(v)$ and $h(x_n) \in h(n)$. Let wRw'. Then $M, w \models_h \Box\phi$, and hence $M, w' \models_h \phi$. Since this holds for every x_n-alternative h of g, we have $M, w' \models_g \forall x_n \phi$. And since this holds for all w' such that wRw', we finally have $M, w \models_g \Box\forall x_n \phi$. □

Completeness

We show that for any ϕ which is not a theorem in CC we can define a CC-model in which ϕ is not valid. The technique we will use in the construction of these models varies only slightly from the standard technique used for modal predicate logic with identity (see in particular (Hughes and Cresswell, 1996)). The worlds of these models will be maximal CC-consistent sets of wffs which have the *witness property*, that is, for all indices n, for all wffs of the form $\exists x_n \phi$, there is a n-indexed variable y_n such that $\exists x_n \phi \rightarrow \phi[x_n/y_n] \in w$. In order to obtain this we will follow the common practice and consider an expanded language $\mathcal{L}^+$, which is $\mathcal{L}_{CC}$ with the addition of a denumerable set of fresh variables.

Before starting I list a number of valid theorems and rules that will be used in the proof, I omit the derivations which are standard. First of all, by ∃**I** and **EGn**, we can derive all standard predicate logic theorems and rules governing the behaviour of quantifiers which do not involve any shift of index, among others the rule of introduction of the universal quantifier ∀**I**, and the principle of renaming of bound variables **PRn**, and the two principles **PL1n** and **PL2n**.

∀**I** $\vdash_{CC} \phi \rightarrow \psi \;\Rightarrow\; \vdash_{CC} \phi \rightarrow \forall x_n \psi$ (provided x_n is not free in ϕ)

PRn $\vdash_{CC} \exists x_n \phi \leftrightarrow \exists y_n \phi[x_n/y_n]$

PL1n $\vdash_{CC} \forall x_n(\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \forall x_n \psi)$ (provided x_n not free in ϕ)

PL2n $\vdash_{CC} \exists z_n(\exists x_n \phi \rightarrow \phi[x_n/z_n])$ (provided z_n not free in $\exists x_n \phi$)

Furthermore, we can prove the following theorem. The derivation uses **ID**, **N** and **SIn**.

LIn $\vdash_{CC} x_n = y_n \rightarrow \Box\, x_n = y_n$

We assume the standard results about maximal consistent sets of wffs with respect to a system S. A set Δ of wffs is *S-consistent* iff there is *no* finite collection $\{\alpha_1, \ldots, \alpha_n\} \subseteq \Delta$ such that $\vdash_S \neg(\alpha_1 \wedge \cdots \wedge \alpha_n)$. A set Δ of wffs is *maximal* iff for every wff α either $\alpha \in \Delta$ or $\neg\alpha \in \Delta$.

THEOREM 3 (Lindenbaum's theorem). *Any S-consistent set of wffs Δ can be enlarged to a maximal S-consistent set of wffs Γ.*

THEOREM 4. *Suppose Γ is a maximal consistent set of wffs with respect to S. Then*

1. *For each wff ϕ, exactly one member of $\{\phi, \neg\phi\}$ belongs to Γ;*
2. *For each pair of wffs ϕ and ψ, $\phi \wedge \psi \in \Gamma$ iff $\phi \in \Gamma$ and $\psi \in \Gamma$;*
3. *If $\vdash_S \phi$, then $\phi \in \Gamma$;*
4. *If $\phi \in \Gamma$ and $\vdash_S \phi \rightarrow \psi$, then $\psi \in \Gamma$.*

We can prove the following theorem.

THEOREM 5. *Let Λ be a consistent set of wffs of $\mathcal{L}_{CC}$. Let $\mathcal{V}_n^+$ be a denumerable set of new variable symbols of index n and let $\mathcal{L}^+$ be the expansion of $\mathcal{L}_{CC}$ formed by adding $\bigcup_{n \in N} \mathcal{V}_n^+$. Then there is a consistent set Δ of wffs of $\mathcal{L}^+$ with the witness property such that $\Lambda \subseteq \Delta$.*

Proof. I follow the standard proof the only difference is that we have not just one sort of variables, but many. We assume that all wffs of the form $\exists v\phi$ for any wff ϕ of $\mathcal{L}^+$, and any variable x_n for each index n are enumerated so that we can speak of the first, the second and so on. We define an increasing sequence of sets of sentences $\Delta_0, \Delta_1, \ldots$ as follows:

$$\Delta_0 = \Lambda$$
$$\Delta_{m+1} = \Delta_m \cup \{\exists x_n\phi \rightarrow \phi[x_n/y_n]\}$$

where $\exists x_n\phi$ is the $m+1$th wff in the enumeration of wffs of that form and y_n is the first variable indexed with n not in Δ_m or in ϕ. Since Δ_0 is in $\mathcal{L}$ and Δ_m has been formed from it by the addition of only m wffs there will be infinitely many n-indexed variables from $\mathcal{V}_n^+$ to provide such a y_n.

We show that if Δ_m is consistent, so is Δ_{m+1}. If this were not the case, then

(i) $\Delta_m \vdash_{CC} \exists x_n\phi$
(ii) $\Delta_m \vdash_{CC} \neg\phi[x_n/y_n]$

Since y_n does not occur in Δ_m, from (ii) by $\forall$**I** we have:

(iii) $\Delta_m \vdash_{CC} \neg\exists y_n\phi[x_n/y_n]$

Now since y_n did not occur in ϕ, $\exists y_n\phi[x_n/y_n]$ is a bound alphabetic variant of $\exists x_n\phi$, and so by **PRn**:

(iv) $\Delta_m \vdash_{CC} \neg\exists x_n \phi$

But (i) and (iv) contradict the consistency of Δ_m. Let Δ be the union of all the Δ_ms. It is easy to see that Δ is consistent and has the witness property. □

Once a set Δ has the witness property each extension of Δ in the same language also has the witness property. Lindenbaum's theorem guarantees that if Δ is consistent there is a maximal consistent set Γ such that $\Delta \subseteq \Gamma$, and so since Δ has the witness property, Γ does too. We can prove the following theorem about maximal consistent sets with the witness property in modal logic.

THEOREM 6. *If Γ is a maximal consistent set of wffs in some language, say $\mathcal{L}^+$, of modal predicate logic, and Γ has the witness property, and α is a wff such that $\Box\alpha \notin \Gamma$, then there is a consistent set Δ of wffs of $\mathcal{L}^+$ with the witness property such that* $\{\psi \mid \Box\psi \in \Gamma\} \cup \{\neg\alpha\} \subseteq \Delta$.

Proof. Again we can use the standard construction (e.g., (Hughes and Cresswell, 1996), pp. 259–261). Again we assume that all wffs of the form $\exists v\phi$ for any wff ϕ of $\mathcal{L}^+$, and any variable x_n for each index n are enumerated so that we can speak of the first, the second and so on. We define a sequence of wffs $\gamma_0, \gamma_1, \gamma_2, \ldots$ as follows:

γ_0 is $\neg\alpha$

γ_{m+1} is $\gamma_m \wedge (\exists x_n \phi \rightarrow \phi[x_n/y_n])$

where $\exists x_n \phi$ is the $m+1$th wff in the enumeration of that form and y_n is the first variable indexed by n such that:

(*) $\{\psi \mid \Box\psi \in \Gamma\} \cup \{\gamma_m \wedge (\exists x_n \phi \rightarrow \phi[x_n/y_n])\}$ is consistent.

In order for this construction to succeed we have to be sure that there always will be a n-indexed variable y_n satisfying (*).

Since γ_0 is $\neg\alpha$, $\{\psi \mid \Box\psi \in \Gamma\} \cup \{\gamma_0\}$ is consistent from a standard result of propositional modal logic. We show that provided $\{\psi \mid \Box\psi \in \Gamma\} \cup \{\gamma_m\}$ is consistent, there always will be a n-indexed variable y_n satisfying (*).

Suppose there were not. Then for every variable y_n in $\mathcal{V}_n^+$, there will exist some $\{\psi_1, \ldots, \psi_k\} \subseteq \{\psi \mid \Box\psi \in \Gamma\}$ such that

(i) $\vdash_{CC} (\psi_1 \wedge \cdots \wedge \psi_k) \rightarrow (\gamma_m \rightarrow \neg(\exists x_n \phi \rightarrow \phi[x_n/y_n]))$

So, by propositional modal logic (**N**, **K** and □-distribution):

(ii) $\vdash_{CC} (\Box\psi_1 \wedge \cdots \wedge \Box\psi_k) \rightarrow \Box(\gamma_m \rightarrow \neg(\exists x_n \phi \rightarrow \phi[x_n/y_n]))$

But Γ is maximal consistent and $\Box\psi_1, \ldots, \Box\psi_k \in \Gamma$, and so

(iii) $\Box(\gamma_m \rightarrow \neg(\exists x_n \phi \rightarrow \phi[x_n/y_n])) \in \Gamma$.

Let z_n be some n-indexed variable not occurring in ϕ or in γ_m. Since Γ has the witness property, then we have for some n-witness y_n:

$$\exists z_n(\neg\Box(\gamma_m \rightarrow \neg(\exists x_n \phi \rightarrow \phi[x_n/z_n]))) \rightarrow \neg\Box(\gamma_m \rightarrow \neg(\exists x_n \phi \rightarrow \phi[x_n/y_n])) \in \Gamma.$$

Since (iii) holds for all $y_n \in \mathcal{V}_n^+$, we then have that:

$$\neg\exists z_n \neg\Box(\gamma_m \rightarrow \neg(\exists x_n \phi \rightarrow \phi[x_n/z_n])) \in \Gamma.$$

But Γ is CC-maximal consistent and hence by **BFn** we have:

$$\Box\forall z_n(\gamma_m \to \neg(\exists x_n\phi \to \phi[x_n/z_n])) \in \Gamma.$$

Since z_n does not occur γ_m, then by **PL1n** we have:

$$\Box(\gamma_m \to \neg\exists z_n(\exists x_n\phi \to \phi[x_n/z_n])) \in \Gamma.$$

But since z_n does not occur in ϕ by **PL2n**, we have

$$\vdash_{CC} \exists z_n(\exists x_n\phi \to \phi[x_n/z_n])$$

but then $\Box\neg\gamma_m \in \Gamma$ and so $\neg\gamma_m \in \{\psi \mid \Box\psi \in \Gamma\}$ which would make $\{\psi \mid \Box\psi \in \Gamma\}\cup\{\gamma_m\}$ inconsistent against our assumption.

Let Δ be the union of $\{\psi \mid \Box\psi \in \Gamma\}$ and all the γ_ms. Since each $\{\psi \mid \Box\psi \in \Gamma\} \cup \{\gamma_m\}$ is consistent, and since $\vdash_{CC} \gamma_m \to \gamma_k$ for $m \geq k$, so is their union Δ. Any maximal consistent extension of Δ has all the required properties and so the theorem is proved. □

I will now show that for each CC-consistent set Δ of wffs of $\mathcal{L}$, we can construct a model M_Δ containing a world in which all the wffs in Δ are true. These models M_Δ are based on *cohesive* sub-frames of the frame $F = \langle W_F, R_F\rangle$ where:

(a) W_F is the set of CC-maximal consistent sets of wffs of $\mathcal{L}^+$ which have the witness property.
(b) wR_Fw' iff for every wff $\Box\phi$ of $\mathcal{L}^+$, if $\Box\phi \in w$, then $\phi \in w'$.

Cohesive frames are frame in which each two worlds are linked by means of some forward or backward R-chain. The reason why we need to consider cohesive models is that in a cohesive model for CC for each index n any world verifies exactly the same identity formulas between variables in $\mathcal{V}_n$.

Let Δ be a CC-consistent set of wffs of $\mathcal{L}_{CC}$. We show how to construct $M_\Delta = \langle W, R, D, I, C\rangle$ in which there is a world w^* such that $\Delta \subseteq w^*$. The constructions of W, R, D, I are standard. C will be some extra work.

W Given some world $w^* \in W_F$ such that $\Delta \subseteq w^*$, we let W be the set of all and only those worlds in W_F which are reachable from w^* by a chain of forward R_F-steps. These worlds are maximal consistent sets of wffs of $\mathcal{L}^+$, which satisfy the witness property.
R R is R_F restricted to W.
D Let $\sim$ be the following relation over the set $\mathcal{V}_0^+$ of variables of $\mathcal{L}^+$ with index 0.

$$v_0 \sim x_0 \text{ iff } v_0 = x_0 \in w.$$

Since W is cohesive (every w, w' in W are linked by some R-chain) and every w contains **LIn** and **LNIn**, it makes no difference which w is selected for this purpose. We can prove the following lemma:

LEMMA 1. $\forall w, w' \in W : \forall n \in N : \forall x, y \in \mathcal{V}^+ :\ x_n = y_n \in w$ *iff* $x_n = y_n \in w'$.

Proof. Consider any two worlds w, w' such that wRw' or $w'Rw$. Suppose $x_n = y_n \in w$. If wRw', then by **LIn**, $\Box x_n = y_n \in w$ and so $x_n = y_n \in w'$. If $w'Rw$, then if $x_n = y_n \notin w'$, then $x_n \neq y_n \in w'$ and so by **LNIn** $\Box x_n \neq y_n \in w'$ and so $x_n = y_n \notin w$, contradicting the assumption. Now since our M_Δ is cohesive, then any two worlds w and w' in W are linked by a chain of backwards or forwards R-steps, and so if $x_n = y_n \in w$ then $x_n = y_n \in w'$, and if $x_n = y_n \in w'$ then $x_n = y_n \in w$. □

It is easy to see that $\sim$ is an equivalence relation (we use **ID** and **SIa** and the maximal consistency of each w). Now for each $x_0 \in \mathcal{V}_0^+$, let

$$[x_0] = \{y_0 \in \mathcal{V}_0^+ \mid x_0 \sim y_0\}$$

be the equivalence class of x_0. We take the domain D to be the set of all these equivalence classes $[x_0]$, for $x_0 \in \mathcal{V}_0^+$ and so define $D = \{[x_0] \mid x_0 \in \mathcal{V}_0^+\}$.

I We now define the interpretation function I for the predicate and individual constant symbols of the language.

(i) For each n-placed relation symbol P in $\mathcal{L}_{CC}$, for each $w \in W$ we define the interpretation $I(P)(w)$ of the symbol P in w as follows: $\forall\langle[x_{0_1}], \ldots, [x_{0_n}]\rangle \in D^n$,

$$\langle[x_{0_1}], \ldots, [x_{0_n}]\rangle \in I(P)(w) \quad \text{iff } Px_{0_1}, \ldots, x_{0_n} \in w.$$

The definition is independent of the representatives of the equivalence classes $[x_{0_1}], \ldots, [x_{0_n}]$, because by **SIa** (or **SIn**) we have

$$\vdash_{CC} Px_{0_1}, \ldots, x_{0_n} \wedge x_{0_1} = y_{0_1} \wedge \cdots \wedge x_{0_n} = y_{0_n} \rightarrow Py_{0_1}, \ldots, y_{0_n}.$$

(ii) Let a be a constant symbol of $\mathcal{L}$ and $w \in W$. From **ID** and **EGa**, we have

$$\vdash_{CC} \exists x_0(a = x_0)$$

So $\exists x_0(a = x_0) \in w$, and because w has the witness property, there is a 0-indexed variable y_0 such that

$$a = y_0 \in w.$$

y_0 may not be unique, but its equivalence class $[y_0]$ is unique because, using **SIa** we have:

$$\vdash_{CC} a = y_0 \wedge a = z_0 \rightarrow y_0 = z_0.$$

The interpretation $I(a)(w)$ in w is this (uniquely determined) element $[y_0]$ of D.

$$I(a)(w) = [y_0] \quad \text{iff } a = y_0 \in w.$$

C Given a variable v_n, let c_{v_n} be the function from W to D such that for all $w \in W$:

$$c_{v_n}(w) = [y_0] \quad \text{iff } v_n = y_0 \in w.$$

The proof that for all w there is such a unique element $[y_0] \in D$ is parallel to the one in the second clause of the definition of I. By **ID** and **EGa**, we have $\vdash_{CC} \exists x_0(v_n = x_0)$ and so $\exists x_0(v_n = x_0) \in w$ for all w, and, therefore, by the witness property of w, $v_n = y_0 \in w$ for some 0-witness y_0. y_0 may not be unique, but its equivalence class $[y_0]$ is, because, by using **SIa**, we have: $\models_{CC} v_n = y_0 \wedge v_n = z_0 \rightarrow z_0 = y_0$.

We let now $CC_n = \{c_{v_n} \mid v_n \in \mathcal{V}_n^+\}$ and we define $C = \{CC_n \mid n \in N\}$.

We have to show that these sets CC_n are conceptual covers.

(i) Existence Condition: $\forall w \in W : \forall[x_0] \in D : \exists c_{v_n} \in CC_n : c_{v_n}(w) = [x_0]$.

Proof. Take any w and $[x_0]$. By **ID** and **EGa**, we have $\exists y_n(y_n = x_0) \in w$ and because w has the witness property we know that $v_n = x_0 \in w$ for some n-witness v_n. Consider now c_{v_n} which is in CC_n. By definition $c_{v_n}(w) = [x_0]$. □

(ii) Uniqueness Condition: $\forall w \in W : \forall c_{v_n} c_{z_n} \in CC_n : c_{v_n}(w) = c_{z_n}(w) \Rightarrow c_{v_n} = c_{z_n}$
Proof. Note firstly that for all $w \in W$ the following holds:

(A) $c_{x_n}(w) = c_{y_n}(w) \;\Leftrightarrow x_n = y_n \in w$.

($\Rightarrow$) by definition of c_{x_n} and c_{y_n}, and **SIa**; ($\Leftarrow$) suppose $x_n = y_n \in w$ by **ID**, **EGa**, **SIa**, and witness property of w, we have for some z_0 and v_0, $x_n = z_0 \in w$, $y_n = v_0 \in w$ and $z_0 = v_0 \in w$, which by definition of c_{x_n} and c_{y_n} means $c_{x_n}(w) = c_{y_n}(w)$.

Suppose now $c_{v_n}(w) = c_{z_n}(w)$ for some $c_{v_n}, c_{z_n} \in CC_n$, and w. By (A) this implies $v_n = z_n \in w$, which, by Lemma 1, implies that for any $w' \in W$, $v_n = z_n \in w'$ and so, again by (A), $c_{v_n}(w') = c_{z_n}(w')$. Since this holds for all $w' \in W$, we have $c_{v_n} = c_{z_n}$. □

We define the *canonical assignment* g as follows: $\forall n \in N, \forall x_n \in \mathcal{V}_n^+$: $g(n) = CC_n$ and $g(x_n) = c_{x_n}$. We can now prove the following theorem:

THEOREM 7. *For any $w \in W$, and any wff $\phi \in \mathcal{L}^+$,*

$$M_\Delta, w \models_g \phi \quad \textit{iff}\ \phi \in w.$$

Proof. The proof is by induction on the construction of ϕ. I start by showing that (B) holds for all t in $\mathcal{L}^+$:

(B) $[t]_{M_\Delta,w,g} = [x_0]$ iff $t = x_0 \in w$.

Suppose t is an indexed variable v_n in $\mathcal{V}_N^+$. Then $[t]_{M_\Delta,w,g} = g(v_n)(w)$. By definition of canonical assignment $g(v_n)(w) = c_{v_n}(w)$. By definition of c_{v_n}, $c_{v_n}(w) = [x_0]$ iff $v_n = x_0 \in w$. Suppose now t is a constant symbol a in $\mathcal{L}$. Then $[t]_{M_\Delta,w,g} = I(a)(w)$. By clause (ii) in the definition of I, $I(a)(w) = [x_0]$ iff $a = x_0 \in w$.

We can now prove the theorem for atomic formulae.

(a) Consider $Rt_1, \ldots, t_n$. Let $w \in W$. By (B), the denotation of the terms $t_1, \ldots, t_n$ in w will be some $[x_{0_1}], \ldots, [x_{0_n}]$ where $t_1 = x_{0_1}, \ldots, t_n = x_{0_n} \in w$. Then

$$\begin{aligned} M_\Delta, w \models_g Rt_1, \ldots, t_n &\Leftrightarrow \langle [x_{0_1}], \ldots, [x_{0_n}] \rangle \in I(R)(w) \\ &\Leftrightarrow Rx_{0_1}, \ldots, x_{0_n} \in w. \end{aligned}$$

But $t_1 = x_{0_1}, \ldots, t_n = x_{0_n} \in w$, thus by various application of **SIa** we have that

$$Rx_{0_1}, \ldots, x_{0_n} \leftrightarrow Rt_1, \ldots, t_n \in w$$

and so $Rx_{0_1}, \ldots, x_{0_n} \in w \;\Leftrightarrow\; Rt_1, \ldots, t_n \in w$.

(b) $M_\Delta, w \models_g t_1 = t_2$ iff $[t_1]_{M_\Delta,w,g} = [t_1]_{M_\Delta,w,g}$. By (B) above this is the case iff $[x_{0_1}] = [x_{0_2}]$ for some $[x_{0_1}]$ and $[x_{0_2}]$ such that $t_1 = x_{0_1} \in w$ and $t_2 = x_{0_2} \in w$. Obviously $x_{0_1} = x_{0_2} \in w$, and therefore by various applications of **SIa** we have that $t_1 = t_2 \in w$.

(c) $M_\Delta, w \models_g \neg\phi$ iff $M_\Delta, w \not\models_g \phi$ iff $\phi \notin w$ iff $\neg\phi \in w$.

(d) $M_\Delta, w \models_g \phi \wedge \psi$ iff $M_\Delta, w \models_g \phi$ and $M_\Delta, w \models_g \psi$ iff $\phi \in w$ and $\psi \in w$ iff $\phi \wedge \psi \in w$.

(e) Suppose $\exists x_n \phi \in w$. By the witness property, for some y_n, we have $\phi[x_n/y_n] \in w$. But then by induction hypothesis $M_\Delta, w \models_g \phi[x_n/y_n]$ which, by standard principle of replacement, implies $M_\Delta, w \models_{g[x_n/g(y_n)]} \phi$. Since $g(y_n) = c_{y_n}$ is an element of $g(n)$ this implies $M_\Delta, w \models_g \exists x_n \phi$.

Suppose $M_\Delta, w \models_g \exists x_n \phi$. Then $M_\Delta, w \models_{g[x_n/c_{v_n}]} \phi$ for some $c_{v_n} \in g(n)$. Since by definition of canonical assignment $g(v_n) = c_{v_n}$, by standard principle of replacement, we have $M_\Delta, w \models_g \phi[x_n/v_n]$. But then, by induction hypothesis, $\phi[x_n/v_n] \in w$ and by **EGn** $\exists x_n \phi \in w$.

(f) Suppose $\Box\phi \in w$ and wRw'. Then $\phi \in w'$ and so $M_\Delta, w' \models_g \phi$. Since this holds for all w' such that wRw', we have $M_\Delta, w \models_g \Box\phi$.

Suppose $\Box\phi \notin w$. Then by Theorem 6 (in combination with Theorem 3) we know that there is some $w' \in W_F$ such that wR_Fw' and $\phi \notin w'$. w' is clearly in W as well since it is accessible from w. Thus by induction hypothesis $M_\Delta, w' \not\models_g \phi$. Since wRw', we can conclude $M_\Delta, w' \not\models_g \Box\phi$. □

THEOREM 8 (Completeness). *If* $\models_{CC} \phi$, *then* $\vdash_{CC} \phi$.

Proof. Suppose $\not\vdash_{CC} \phi$. Then $\neg\phi$ is CC-consistent. We then know that $\neg\phi$ is an element of some world w of the model $M_{\{\neg\phi\}}$ generated by $\{\neg\phi\}$ and, therefore, by Theorem 7 true in w in that model. This means that $M_{\{\neg\phi\}} \not\models_{CC} \phi$, and, therefore, $\not\models_{CC} \phi$. □

NOTES

[1] See (Quine, 1953, 1956, 1960) 'What is this object, that denounced Catiline without Philip's having become aware of the fact?' (Quine, 1953, p. 147).

[2] See in particular (Hintikka, 1967, 1969), but also (Kraut, 1983) and more recently (Gerbrandy, 2000).

[3] The labels used for the principles in this section are historically motivated. See, for example, (Hughes and Cresswell, 1996) for the relevant references.

[4] Numerous objections have been raised against the intuitive validity of these two principles. The standard way to provide a semantics where the Barcan formula does not hold is to allow for models with increasing domains. In order to do so, a model is defined as a quintuple: $\langle W, R, D, F, I\rangle$ where W, R, D, I are as above, and F is a function from W to subsets of D, which satisfies the following condition: if wRw', then $F(w) \subseteq F(w')$. If we want to falsify the converse of the Barcan formula, we need to drop the inclusion requirement (see (Hughes and Cresswell, 1996) for a clear formal discussion of these issues). However, since considerable difficulties arise if we drop the inclusion requirement, and since the philosophical issue related to the intuitive interpretation of the Barcan formula and its converse is not prominent in the present work, I will restrict my discussion to a semantics in which domains are not allowed to vary.

[5] Substitution of co-referential terms and existential generalization are allowed, if applied to variables or in the absence of any belief operator. The following principles are valid in MPL:

SI1 $t_1 = t_2 \to (\phi[t_1] \to \phi[t_2])$ (if ϕ is non-modal)

SIv $x = y \to (\phi[x] \to \phi[y])$

EG1 $\phi[t] \to \exists x\phi[x]$ (if ϕ is non-modal)

EGv $\phi[y] \to \exists x\phi[x]$

[6] A weaker version of the principle of substitutivity of identicals holds for sentences containing a belief operator, if we assume consistency, positive and negative introspection:

$$\mathbf{SI}_{\Box} \quad \Box t_1 = t_2 \rightarrow (\Box\phi[t_1] \rightarrow \Box\phi[t_2])$$

If we are discussing what a person believes we can substitute a term for another if they refer to one and the same individual in all her doxastic alternatives. If we consider all models rather than only serial, transitive, and Euclidean models, the principle holds only if ϕ is non-modal.

[7] It is important to notice that the phenomena which are typically considered in discussions of rigid designators (alethic modalities and counterfactuals) are of a different nature than the epistemic phenomena considered here. Many authors (e.g., Hintikka, 1975; Bonomi, 1983) have distinguished semantically rigid designators from epistemically rigid designators – the former refer to specific individuals in counterfactual situations, the latter identify objects across possibilities in belief states – and concluded that proper names are rigid only in the first sense.

[8] Example (20) follows by simple reasoning. Example (21) is derived by substitution of co-referential terms which holds if operated outside the scope of a belief operator (with the assumption that Putin is the president of Russia).

[9] If we consider also non-serial, non-transitive and non-Euclidean models, the two principles are valid only if ϕ does not contain any belief operator.

[10] MPL also validates the following scheme:

$$\mathbf{LNIv} \quad x \neq y \rightarrow \Box x \neq y$$

And in all serial models also the following is valid:

$$\mathbf{CLIv} \quad \Box x = y \rightarrow x = y$$

LNIv and **CLIv** are not discussed by Church, since they are not derivable by simple application of substitutivity.

[11] If there are no worlds accessible from w, $\Box x \neq y$ is true in w, even if x and y refer everywhere to one and the same individual.

[12] This is a quote from (Church, 1982, p. 62), who quotes (Quine, 1953, p. 151).

[13] This question is structurally identical to the initial question of this article.

[14] This use of individual concepts can be seen to be anticipated in (Frege, 1892) and (Carnap, 1947).

[15] Quine considers intensions (individual concepts but also propositions) 'creatures of darkness' (Quine, 1956, p. 180) and analyzes his notional belief reports as relations between individuals and sentences rather than propositions. I will disregard this issue here.

[16] 'The kind of exportation which leads from (30) to (31) should doubtless be viewed in general as implicative' (Quine, 1956, p. 182).

[17] The quotation is from the end of (Quine, 1961).

[18] '...a solution might lie in somehow picking out certain kind of names as being required for the exportation' (Kaplan, 1969, p. 221).

[19] Kaplan (1969) follows Quine (1956) in assuming that objects of belief are sentences and not propositions, but by using Frege's method of representation of intermediate contexts he manages to account for the *de re–de dicto* ambiguity by permutation of scope rather than by positing two primitive senses of beliefs.

[20] If we consider multi-modal extensions of the semantics, different subjects can be taken to have different conceptual repertoires. So in order to properly extend Kaplan's analysis to

these models, we should take different sets of suitable concepts S_a as assigned to different agents $a \in A$.

[21] 'Indeed, Leo has a 'name' α of Pio (namely 'the man wearing a bush jacket at the theater') such that: (i) α actually denotes Pio; (ii) α is in a causal relation with Pio (since it is originated in a perceptive contact); (ii) and α is a sufficiently vivid name, for Leo, of Pio (again because of the perceptive contact). In addition, Leo believes that the man wearing a bush jacket at the theater has climbed the Cervino mountain.' (Bonomi, 1995, my translation.) Therefore, at least if we assume Kaplan's characterization of the notion of a suitable representation, condition (A) is satisfied in this case.

[22] Such an ambiguity does not seem to have been empirically observed before the above-mentioned philosophical theory has been proposed. It seems an unexpected consequence of the theory rather than a meant prediction. Indeed, as far as I know, nobody has ever argued in favor of it.

[23] That the context of utterance (in particular the intentions of the participant in the conversation) is relevant for the interpretation of *de re* belief attributions has been observed among others by van Fraassen (1979), Stalnaker (1988) and Crimmins and Perry (1989), and more recently, again, in (van Rooy, 1997) and (Gerbrandy, 2000).

[24] See Westerståhl (1984) who also defends the view that quantificational domains are pragmatically determined.

[25] In order to avoid this counterintuitive result, a proponent of CIB can either assume that the set of suitable concepts S satisfies the existence condition or it can relativize the interpretation function I to S. The latter strategy is obviously not available in a pragmatic approach, where the choice of I is prior to the contextual selection of Z.

[26] By standard techniques we can show that there are exactly $(|D|!)^{|W|-1}$ conceptual covers based on (W, D).

[27] Note that Kripke says that the two sentences are intuitively true in their *de dicto* interpretation and still should not imply that Pierre's beliefs are inconsistent. However, given our intuitive characterization of *de dicto* belief this does not seem correct. Indeed, we could say that (65a) results from an application of **SI** from the sentence:

(67) Pierre believes that Londres is pretty.

where 'London' and 'Londres' are co-referential terms belonging to different languages. But *de dicto* belief does not seem to allow **SI** even if the two co-referential terms are part of different languages.

[28] Again we must assume consistency, positive and negative introspection. If we consider also non-serial, non-transitive and non-Euclidean CC-models, the two principles are valid only if ϕ does not contain any belief operator.

[29] The pragmatic selection of a conceptual perspective seems then to be ruled by principles which are not absolute, but may be crucially violated in order to prevent the violation of more stringent ones. This suggests that a formalization of these procedures should utilize the framework of Optimality Theory. See (Aloni, 2003) for such a formalization.

[30] The content under discussion can be conveyed by the *de dicto* reading of sentence (85) without any shift in the domain of quantification. Therefore sentence (85) is more cooperative than (84), which, to convey the same content, requires such a shift.

[31] Similar blocking effects have been used to explain phenomena of disparate nature by many authors, in particular Larry Horn and more recently Reinhard Blutner.

[32] In the given contexts, the content under discussion could have been expressed by the *de dicto* reading of (93) without any shift in the domain of quantification.

[33] In Bonomi's example (39) of Odette's lover, the accommodation of a non-rigid domain is justified, as in the case of Susan's mother, by the fulfillment of the Referential Device Principle. The chosen referential device 'the chief of the army' is a better option for the participants to that conversation than the alternative 'Odette's lover' which among other inadequacies involves a proper name which requires familiarity.

[34] As I have argued in (Aloni, 2002), 'knowing who' constructions show the same context dependence as *de re* belief attributions, their interpretation being relative to the operative method of cross-identification. So, on the present account, the sentence 'He knows who Susan's mother is' can obtain different interpretations depending on the adopted conceptualization. The only interpretation which would make the sentence true in the described context is one which accommodates a domain containing the concept $\lambda w[\text{Susan's mother}]_w$. This interpretation would trivialize the sentence though, and, therefore, is pragmatically incorrect.

[35] Hughes and Cresswell (1996, pp. 354–356) show a similar result, namely that Lewis' counterpart theory and Modal Predicate Logic define the same notion of validity if the counterpart relation C is assumed to satisfy the following conditions: (a) C is an equivalence relation; and (b) an individual has one and only one counterpart in each world. As it is easy to see, conceptual covers and counterpart relations which satisfy these two conditions flesh out exactly the same notion.

[36] I am indebted to Paul Dekker for simplifying my original version of this construction.

[37] This axiomatization is based on the axiom system of modal predicate logic with identity in (Hughes and Cresswell, 1996). See in particular Chapters 13, 14 and 17.

[38] **SI1** may be deduced from **SIa** by induction on the construction of $\phi[t_1]$ and $\phi[t_2]$ (the proof is standard). From **SI1** we can derive **EG1** as follows (for ϕ non-modal):

(1) $\vdash_{CC} t = x_n \rightarrow (\phi[t] \rightarrow \phi[x_n])$	**SI1**
(2) $\vdash_{CC} t = x_n \rightarrow (\phi[t] \rightarrow \exists x_n \phi[x_n])$	(1)×**EGn**×**PC**
(3) $\vdash_{CC} \exists x_n (t = x_n) \rightarrow (\phi[t] \rightarrow \exists x_n \phi[x_n])$	(2)×∃**I**
(4) $\vdash_{CC} \exists x_n (t = x_n)$	**ID**×**EGa**×**MP**
(5) $\vdash_{CC} \phi[t] \rightarrow \exists x_n \phi[x_n]$	(4)×(3)×**MP**

[39] $\mathbf{SI}_\Box$ may be deduced from **SI1**, **N** and **K**. From $\mathbf{SI}_\Box$, we may derive $\mathbf{EG}_{\Box n}$ as follows (for ϕ non-modal):

(1) $\vdash_{CC} \Box t = x_n \rightarrow (\Box\phi[t] \rightarrow \Box\phi[x_n])$	$\mathbf{SI}_\Box$
(2) $\vdash_{CC} \Box t = x_n \rightarrow (\Box\phi[t] \rightarrow \exists x_n \Box\phi[x_n])$	(1) ×**EGn**× **PC**
(3) $\vdash_{CC} \exists x_n \Box t = x_n \rightarrow (\Box\phi[t] \rightarrow \exists x_n \Box\phi[x_n])$	(2) ×∃**I**

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