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Core Logic
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Exercise 18

1) i = infinity, because the limits of A and T as they go to infinity approach 2.
Achilles is still at 2 at time infinity + infinity,
because infinity + i = infinity again.

2)
n = 0. T_0 and T^*_0 are both defined to be 1.
$A_1 = A_0 + |T_0 - A_0|$
$A_1 = 0 + |1 - 0| = 1$
$A^*_1 = A_0 + 1/(2^{v(0)})$
$A^*_1 = 0 + 1/1 = 1$

n = 1.
$T_1 = T_0 + |T_0 - A_0|$
$T_1 = 1 + 1/2 * |1 - 0| = 1.5$
$T^*_1 = T_0 + 1/(2^{v(1)})$
$T^*_1 = 1 + 1/2 = 1.5$

$A_2 = A_1 + |T_1 - A_1|$
$A_2 = 1 + |1.5 - 1| = 1.5$
$A^*_2 = A_1 + 1/(2^{v(1)})$
$A^*_2 = 1 + 1/2 = 1.5$

And so on. The formulas can be described by this single formula
(except that for Achilles the sequence is prefixed with 0):

Let i = 1 till infinity: $\mathrm{Sum}(1/(2^i))$

$A^*_{\text{infinity}+5} = 2 + 1 + 0.5 + 0.25 + 0.125 + 0.0625 = 3.9375$
First we take the result of A^*_{infinity}, which converges to 2. Then we re-iterate the cycle 5 times to arrive at infinity + 5.

$T^*_{\text{infinity}+12} = 2 + 0.5 + 0.25 + 0.125 + 0.0625 + \ldots + 0.00024 = 2.9997\ldots$
Same as before, but now we skip $1/2^0=1$ because the turtle's formula increments the value v(i) by one.

$A^*_{\text{infinity}+\text{infinity}} = 4$, Achilles is at 4 at time infinity + infinity, because once Achilles passes the Tortoise, the sequence starts again and converges to 2 again, and 2 + 2 = 4.
$T^*_{\text{infinity}+\text{infinity}} = 3$, similar to Achilles, but since the turtle skips the step with $1/2^0 = 1$, the turtle is behind. This makes sense because the turtle is slower than Achilles by a factor of 1/2.

Exercise 19

We take L* to be an extension of L, with a new constant .c.
We add the relation greater than to our language:
x > y := there is a z: z =/= 0 ^ x = y + z

We define S as the set containing the following formulas:
.c > 0
.c > 1
.c > 1 + 1
.c > (1 + 1) + 1
...

So .c is bigger than all the natural numbers, because it is bigger than all the successors of zero. Let T' = T U S. Every finite subset of T' can be made true in some model M, because it contains some sentences of T, and a finite amount of sentences of the form .c > k. We can interpret .c as a number larger than the largest k such that .c > k is in the subset.

Now by the compactness theorem it follows that the whole set T' is true in some model M. This model M will contain all the natural numbers, because of T, but it also contains .c, which is not a natural number, thus it is not isomorphic to N.

Exercise 20

1) Consistent. Model: I(R) = { <1,2>, <1,3>, <2,3> }
2) Consistent. Model: I(R) = {}
3) Inconsistent. If we assume a domain with at least two individuals, then phi_ii requires there to be a relation between them. This relation satisfies the antecedent of phi_LEP, which means ~phi_ME is contradicted.
We must assume a domain with at least two individuals because of phi_ME. Suppose we have a domain with one individual, then R does not hold for this individual because of phi_i, but this individual will be equal to all other individuals in the domain. However, ~phi_ME says that neither may hold.

Exercise 21

1) Because with the axiom of separation we have to start from a specific set. With the axiom of comprehension we were able to implicitly start from the universal set. Using the axiom of separation it is not possible to arrive at the universal set, because this axiom is much more restricted, as is shown by the need for ZFC to add axioms for the existence of the union and powerset of sets.

2) We substitute the universal set V for X in the axiom of separation:
R = {z in V : phi(z)} where phi(z) = z not in z
To obtain the Russell set R. Now let's ask whether R is in R:
Suppose R is not in R, then by phi it should be in R.
Suppose R is in R, then by phi it shouldn't be in R.