UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2009/2010; 1st Semester
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Homework Set # 2 *Deadline:* September 29, 2009

Exercise 5 (10 points).

We return to the world of sheep and owners of Exercise 3. The following are two possible formalizations of the 'birth cycle' of Exercise 3:

Formalization 1. Let P, S be finite pairwise disjoint sets (representing 'people', 'sheep'). Let $B \subseteq S$ be the set of 'black sheep' and $W \subseteq S$ the set of 'white sheep'. Let $O \subseteq S \times P$ be the ownership relation. We define the semantics for our syntactic objects as follows: $\mathsf{black}(s)$ iff $s \in B$; $\mathsf{white}(s)$ iff $s \in W$; $\mathsf{owner}(p, s)$ iff $\langle s, p\rangle \in O$.
We call $\langle P, S, B, W, O\rangle$ an *ovine model* if $W \cap B = \emptyset$ and $W \cup B = S$ and if for all $p, q \in P$ and $s \in S$, we have that if $\langle s, p\rangle \in O$ and $\langle s, q\rangle \in O$, then $p = q$.
Suppose that $S = S_0 \cup S_1$ where $S_0 \cap S_1 = \emptyset$ (representing 'old sheep' and 'new sheep'). A relation $R \subseteq S_0 \times S_1$ is called a *birth relation* if it has the following properties:

- If $s \in S_0 \cap B$ and $\langle s, t\rangle \in R$, then $t \in S_1 \cap B$.
- If $s \in S_0 \cap W$ and $\langle s, t\rangle \in R$, then $t \in S_1 \cap W$.
- If $\langle s, p\rangle \in 0$ and $\langle s, t\rangle \in R$, then $\langle t, p\rangle \in O$.
- For every $t \in S_1$ there is exactly one $s \in \S_0$ such that $\langle s, t\rangle \in R$.

We say that an ovine model is an *implicit birth cycle model* if there is a partition $S = S_0 \cup S_1$ such that S_0 has exactly twice as many elements as S_1 and there is a birth relation $R \subseteq S_0 \times S_1$. We interpret $\langle s, t\rangle \in R$ as "s gives birth to t".

Formalization 2. Let $P, S = S_0 \cup S_1, B, W, O$ be as in Formalization 1. We call a model $\langle P, S_0, S_1, B, W, O\rangle$ an *explicit birth cycle model* if

- $\langle P, S_0 \cup S_1, B, W, O\rangle$ is an ovine model,
- R is a birth relation
- S_0 has exactly twice as many elements as S_1.

Describe briefly the difference between Formalization 1 and Formalization 2. Pay particular attention to the question whether a relation M ('is a mother of') can be defined in implicit or explicit birth cycle models. (2 points)

Now suppose you are told that "10% of all sheep die of old age" after the birth cycle. Which of the two formalizations would you choose as the starting point to formalize this statement and why? (1 point). Would this change if the additional information was "10% of all *old* sheep die of old age"? Discuss. (1 point).

Give a precise definition of a class of models that you call *birth and old age models* and that capture "10% of all sheep die of old age". (2 points). Give an example of a birth and old age model in which the conclusion of Exercise 3 is violated, i.e., no shepherd owns only white sheep at the beginning, but there is a shepherd who owns only white sheep after the birth and old age steps. (1 point).

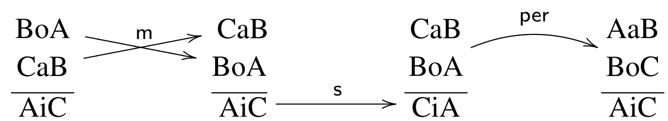
Now suppose that instead of old age, you have the additional information "25% of all sheep giving birth die". Which of the two formalizations would you now choose as the starting point to formalize this statement and why? (1 point). Give a precise definition of a class of models that you call *birth and death models* and that capture this information. (2 points).

Exercise 6 (6 points).

Let $\mathfrak{B}_{\mathrm{GH}} := \{\mathbf{Giliri}, \mathbf{Halodri}\}$

where **Giliri** is
$$\begin{array}{c} \mathrm{AiB} \\ \mathrm{BiC} \\ \hline \mathrm{AiC} \end{array}$$
and **Halodri** is
$$\begin{array}{c} \mathrm{AaB} \\ \mathrm{BoC} \\ \hline \mathrm{AiC} \end{array}.$$

For example, the following is a $\mathfrak{B}_{\mathrm{GH}}$-proof:



Following the proof, the mood $\mathrm{BoA}, \mathrm{CaB}{:}\mathrm{AiC}$ could be called **Homalis**.

Give $\mathfrak{B}_{\mathrm{GH}}$-proofs in the graphic representation and find names consistent with the medieval mnemonics for the following three moods: (2 points each)

$$\begin{array}{c} \mathrm{AeB} \\ \mathrm{CoB} \\ \hline \mathrm{AoC} \end{array} \qquad \begin{array}{c} \mathrm{BiA} \\ \mathrm{BaC} \\ \hline \mathrm{AiC} \end{array} \qquad \begin{array}{c} \mathrm{BeA} \\ \mathrm{CaB} \\ \hline \mathrm{AiC} \end{array}.$$

Exercise 7 (4 points).

A categorical proposition is called *particular* if it has 'i' or 'o' as a copula. Let M be a mood such that both premises of M are particular. Argue that $\mathrm{BCDF} \nvdash M$.

Exercise 8 (5 points).

Read

> Alan Code, "Aristotle's Response to Quine's Objections to Modal Logic", *Journal of Philosophical Logic* 5 (1976): pp. 159–186.

and answer the following questions.

(1) Paraphrase Smullyan's solution to the problem of "The president resigned last August" in one sentence. (1 point)

(2) Does Code believe that Aristotle had something like Smullyan's solution in mind? (Give a brief argument; 2 points)

(3) Explain briefly (at most 100 words) what Code means when he says "Ford is not a spatio-temporal worm but rather . . . a hydra". (2 points)