
BASIC LOGIC, HOMEWORK 2

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Exercise 1 Make natural deductions which show that:

- (a) $\neg p \vdash p \rightarrow q$
- (b) $q \vdash p \rightarrow q$
- (c) $\neg p \vee q \vdash p \rightarrow q$
- (d) $p \vee q, \neg p \vdash q$
- (e) $\neg(p \vee q) \vdash \neg p \wedge \neg q$
- (f) $\neg p \vee \neg q \vdash \neg(p \wedge q)$
- (g) $\neg p \wedge \neg q \vdash \neg(p \vee q)$
- (h) $p \wedge q \vdash \neg(\neg p \vee \neg q)$
- (i) $p \wedge \neg q \vdash \neg(p \rightarrow q)$
- (j) $p \vee q \vdash (p \rightarrow q) \rightarrow q$

Exercise 2 The next ones look no more difficult than the above, but they are.

- (a) $p \rightarrow q \vdash \neg p \vee q$
- (b) $\neg(p \wedge q) \vdash \neg p \vee \neg q$
- (c) $\neg(p \rightarrow q) \vdash p \wedge \neg q$
- (d) $(p \rightarrow q) \rightarrow q \vdash p \vee q$

Exercise 3 Make natural deductions which show that:

- (a) $\neg q \vee p, q \wedge r, \neg p \vdash \neg r$
- (b) $p \wedge q, p \rightarrow (r \rightarrow s), q \rightarrow (r \rightarrow \neg s) \vdash \neg r$
- (c) $p \rightarrow (q \vee r), q \rightarrow r \vdash \neg r \rightarrow \neg p$
- (d) $(q \rightarrow p) \rightarrow r, (p \rightarrow r) \rightarrow q \vdash q$
- (e) $\vdash (\neg \perp \rightarrow \perp) \rightarrow (\neg \perp \wedge \perp)$
- (f) $(p \rightarrow p) \rightarrow q \vdash p \rightarrow q$
- (g) $p \rightarrow \neg r, \neg p \rightarrow \neg r \vdash \neg r$
- (h) $\neg q \rightarrow \neg(p \rightarrow q), \neg p \vdash q$
- (i) $\neg p \wedge r \vdash \neg((p \rightarrow \neg r) \rightarrow \neg r)$
- (j) $\neg(\neg r \rightarrow s) \rightarrow p, \neg r \vdash p \vee s$
- (k) $(p \wedge q) \rightarrow r \vdash (p \rightarrow r) \vee (q \rightarrow r)$

Exercise 4 Only for the addicted:

- (a) $\neg(p \wedge r), p \wedge q, q \rightarrow (s \vee r) \vdash s$
 - (b) $p \rightarrow (q \vee r), (\neg q \wedge t) \vee (s \rightarrow p), \neg(\neg r \rightarrow \neg p) \vdash \neg s \vee q$
 - (c) $(p \rightarrow q) \vee r, s \rightarrow t, \neg(q \vee r) \vee \neg t \vdash \neg p \vee \neg s$
 - (d) $(p \vee q) \rightarrow (r \vee s), \neg(s \rightarrow \neg p) \rightarrow q, \neg(p \rightarrow r) \vdash p \wedge q$
 - (e) $(p \wedge q) \rightarrow \neg r, r \vee (s \wedge t), (p \wedge q) \vee (\neg p \wedge \neg q) \vdash p \rightarrow s$
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